

**Homework 08, Morally Due 12:30PM, Tue Apr 7 2026**

1. (0 points) What is your name?

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2. (30 points) Let  $\text{COL}: \binom{[n]}{2} \rightarrow [\omega]$ . Assume that, for all colors  $c$ , for all  $x \in [n]$ ,  $\deg_c(x) \leq 17$ . Show that there is a rainbow set of size  $\Omega(n^{1/3})$ .

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3. (40 points) **Recall:** we showed the following:

For all  $f: Z \times Z \rightarrow Z$  there exists infinite  $D \subseteq Z$  such that  $f$  restricted to  $D \times D$  is NOT onto.

Recall that we used the 2-ary Ramsey Theory THREE times.

And NOW for our problems

(a) Let  $f: Z \times Z \times Z \rightarrow Z$ .

Show there exists a  $D \subseteq Z$  such that

$f$  restricted to  $D \times D \times D$  is not onto.

How many times does your proof use the 3-ary Ramsey theorem?

(b) Let  $a \in \mathbf{N}$ .

Let  $f: Z^a \rightarrow Z$ .

Show there exists a  $D \subseteq Z$  such that

$f$  restricted to  $D^a$  is not onto.

Let  $g(a)$  be the number of times your proof uses the  $a$ -ary Ramsey theorem.

If you did the problem correctly then  $g(a)$  does not have a nice closed form but it is the solution to a combinatorial problem. What is that problem?

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4. (30 points) **Recall** that a finite set  $X$  is *Large* if  $|X| > \min(X)$ .

Prove the following:

For all  $k$ , there exists an  $n$ , such that, for all  $\text{COL}: \binom{\{k, k+1, \dots, k+n\}}{2} \rightarrow [\omega]$  there is a large set that is either

- *homog*
- *min-homog*
- *max-homog*
- *rainbow*

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5. (0 points, but you must answer it)

As you all know, Yasmine's stage name is *Bad Sequence*.

If I had a rock band I would call it *The Red Cliques*.

A grad student in quantum computing had a group called *Amplitudes with Attitude*.

Dave Mount (Geometry) would name his group *Fried Green Ellipsoids*.

The following are real groups:

*The Mobius Band*,

*The Klein Four*,

*The MIT Logarithms*.

There is a rap singer named MC++. There is subgenre of rap called *Nerdcore* where people sing about math and technology things. *Drama in the PhD* is the best example of that. Do not confuse this with *mathcore* which is a *highly technical, chaotic subgenre of hardcore punk and metalcore* which in my opinion is just noise.

OKAY, so finally here is the question:

If you had a stage name, what would it be?

If you had a rock group, what would you name it?