Homework 12, Morally Due 12:30PM, Tue May 5 2026

1. (0 points) What is your name?

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2. (30 points)

(a) (0 points) Write a program that, on input (n, k, c), looks at ALL c-colorings of $[n]$, counts how many HAVE a k-AP, and divides that by c^n. Hence you get the fraction of c-colorings that have a mono k-AP to 3 places.

(b) (15 points) For $3 \le n \le 7$ determine what fraction of 2-colorings of $[n]$ have a mono 3-AP. Output a table like the one below. THE NUMBERS WE GIVE ARE FALSE or true by accident.

n	Fraction of 2-colorings of $[n]$ that have a mono 3-AP
3	0.5
4	0.617
5	0.689
6	0.75
7	0.81
8	0.999

(c) (15 points) For $3 \le n \le 34$ determine what fraction of 2-colorings of $[n]$ have a mono 4-AP. Output a table similar to the one in the last problem.

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3. (40 points. This problem is also on the optional project so if you doing the optional project then use your answer for both this HW and the project.)

   Let $VDW(k, \omega)$ mean that VDW theorem is known for k-APs and ANY number of colors.

   Assume you know VDW theorem for $VDW(1, \omega)$, $VDW(2, \omega)$, $VDW(3, \omega)$, and $VDW(4, \omega)$. Prove VDW theorem for $VDW(5, 2)$.

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4. (30 points. This problem is also on the optional project so if you doing the optional project then use your answer for both this HW and the project.) You cannot Use Rado's Theorem for this problem. We will be asking you to prove Rado's Theorem in this particular case. You CAN use the Extended VDW theorem.

   Consider the equation

$$E: \quad w + 2x - 3y + z = 0.$$

   Show that there exist a number M such that, for all $\mathrm{COL}\colon [M] \to [2]$, there exists a mono solution to E with w, x, y, z all distinct.