752 Midterm for Spring 2026

1. (0 points) What is your name

GO TO NEXT PAGE

2. (25 points) Prove that, for every c, for every COL: $\binom{\mathsf{N}}{2} \rightarrow [100]$, there is an infinite homog set.

   You have to prove this from scratch. You can't invoke Ramsey's Theorem. You CAN just show me the first few steps of the construction and say the rest is clear if the rest really is clear.

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3. (25 points)

(In this problem you may use the a-ary Ramsey Theorem for any $a \geq 1$.)

Definition Let $\mathrm{COL}_1 \colon \mathsf{N} \to [10]$. Let $\mathrm{COL}_2 \colon \binom{\mathsf{N}}{2} \to [10]$.

Let H be an infinite subset of N.

- H is *super-dooper homog* if there is a color c such that
  $\mathrm{COL}_1 \colon H \to [10]$ ONLY uses the color c and
  $\mathrm{COL}_2 \colon \binom{H}{2} \to [10]$ ONLY uses the color c
  (For example, COL_1 colors every element of H RED, and COL_2 colors every pair of elements of H RED.)
- H is *super homog* if there are two colors c_1, c_2 (they could be the same but do not have to be)
  such that
  $\mathrm{COL}_1 \colon H \to [10]$ ONLY uses the color c_1 and
  $\mathrm{COL}_2 \colon \binom{H}{2} \to [10]$ ONLY uses the color c_2
  (For example, COL_1 colors every element of H RED, and COL_2 colors every pair of elements of H BLUE.)

End of Definition

And now for the questions!

(a) Show that, for all
$\mathrm{COL}_1 \colon \mathsf{N} \to [10]$ and $\mathrm{COL}_2 \colon \binom{\mathsf{N}}{2} \to [10]$,
there exists an infinite super-homog set.

(b) Show that, there exists a
$\mathrm{COL}_1 \colon \mathsf{N} \to [10]$ and $\mathrm{COL}_2 \colon \binom{\mathsf{N}}{2} \to [10]$
such that there is no super-dooper set.

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4. (25 points) Show that if $(X, \preceq_X)$ and $(Y, \preceq_Y)$ are wqo then $(X \times Y, \preceq)$ is a wqo where $(x_1, y_1) \preceq (x_2, y_2)$ iff $((x_1 \preceq_X x_2)$ and $(y_1 \preceq_Y y_2))$
You may use Ramsey's Theorem.

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