BILL, RECORD LECTURE!!!!

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When Does a 2-Coloring Yield a Mono Unit Square?

Exposition by William Gasarch

April 7, 2026

Credit Where Credit is Due!

The main theorem of these slides is due to
Paul Erdös, Ronald Graham, Peter Montgomery, Bruce L. Rothchild, Joel Spencer, Ernst G. Straus.

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Journal of Combinatorial Theory (A), Vol. 14, 341-363, 1973

https://www.cs.umd.edu/~gasarch/TOPICS/eramsey/eramseyOne.pdf

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Question Is there a proper 2-coloring of $\mathbb{R}^2$?

Answer Yes. We leave this for an exercise.

What About Higher Dimensions?

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The problem is open in $\mathbb{R}^3$.

Darling That's to bad. We live in $\mathbb{R}^3$.

We Will Prove. . .

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$f_1 = (\frac{1}{\sqrt{2}}, 0, 0, \dots, 0)$ 9 coordinates,
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Example
$e_2 f_7 = (0, \frac{1}{\sqrt{2}}, 0, 0, 0, 0, 0, 0, 0, \frac{1}{\sqrt{2}}, 0, 0)$.

Lemma Left To Reader

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Lemma Let $1 \leq i < j \leq 3$ and $1 \leq k < \ell \leq 9$.
Then the following points form a unit square as shown:

$$\begin{matrix} e_i f_\ell & & e_j f_\ell \\ & & \\ e_i f_k & & e_j f_k \end{matrix}$$

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Thm For all c, m, there exists n such that, forall $\mathrm{COL}\colon \mathbb{R}^n \rightarrow [c]$ there exists a mono unit m-cube.

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Two of them are the same color and are a copy of X.

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Note We wanted an n such that $\forall \mathrm{COL}\colon \mathbb{R}^n \rightarrow [c]$ there would be a mono X.

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We won't prove this.

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We showed that, for all m, the unit m-cube is Ramsey.

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Obtaining More Ramsey Sets

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Thm If X and Y are Ramsey then $X * Y$ is Ramsey.