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Finite Ramsey Theorem For 3-Hypergraph

Exposition by William Gasarch

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The Finite 3-Hypergraph Ramsey Theorem

Thm $(\forall k)(\exists n)$ such that $(\forall)\mathrm{COL}\colon \binom{[n]}{3} \rightarrow [2]$ there exists a homog set of size k.

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We do an example of the first few steps of the construction.

First Step of Our Construction

Since every 3-subset has a color, harder to draw pictures so I won't :-(.

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$\mathrm{COL}(1, 2, 3) =$ **R**.
$\mathrm{COL}(1, 2, 4) =$ **B**.

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$\mathrm{COL}(1,2,5) =$ **B**.

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$\mathrm{COL}(1, 3, 4) =$ **R**.
$\dots$

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$\dots$
$\mathrm{COL}(1, n-1, n) = \mathbf{B}$.

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$\mathrm{COL}(1, n-1, n) =$ **B**.

What to make of this?

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$\mathrm{COL}(1,n-1,n) = $ **B**.

What to make of this? Discuss.

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Use the following approximation to the truth:
For all $\mathrm{COL}\colon \binom{[n]}{2} \rightarrow [2]$ there exists a homog set of size $\geq \log n$.