Definitions

1. VC_k is the set of graphs that have a vertex cover of size $\le k$.

2. OBS_k is an obstruction set for VC_k. OBS_k is known to exist by the GMT. By definition of *obstruction set* we know that

$$G \in VC_k \text{ iff } (\forall H \in \mathrm{OBS}_k)[H \not\preceq_{\mathrm{m}} G].$$

(We take OBS_k to mean any finite obstruction set for VC_k. Usually OBS_k means the minimal one which is unique.)

Algorithm 1 We have an $O(|E|) = O(n^2)$ algorithm for:
Given $G = (V, E)$ and a set $U \subseteq V$, determine if U is a vertex cover.

Algorithm 2 We have an $O(n^k)$ algorithm for:
Given G determine if it is $\notin \mathrm{VC}_k$ or in $\mathrm{VC}_k - \mathrm{VC}_{k-1}$ or in $\ldots$ $VC_2 - \mathrm{VC}_1$.

Algorithm 3 *Let H be a graph. We have an $O(g(H)n^3)$ algorithm for:*
Given G, determine if $H \preceq_{\mathrm{m}} G$.
(Given H we can obtain the code very quickly.)

Algorithm 4 *If we have OBS_k, then we have an FPT algorithm for:*
Given G, does it have a vertex cover of size $\le k$ (we've called this VC_k).
Whiggish *We can run Algorithm 4 with a finite set $\mathrm{PHOBS}_\mathbf{k}$ of graphs that are not in VC_k, but may not form OBS_k. If we find some $H \in \mathrm{PHOBS}_k$ with $H \preceq_{\mathrm{m}} G$ then $G \notin \mathrm{VC}_k$.*

Algorithm 5 *If we have $\mathrm{OBS}_1, \ldots, \mathrm{OBS}_k$ then we have an FPT alg for:*
Given a graph G find if it has a VC of size k and if it does then output the actual VC
Whiggish *We can run Algorithm 5 with finite sets $\mathrm{PHOBS}_\mathbf{1}$,..., $\mathrm{PHOBS}_\mathbf{k}$ of graphs that might not be $\mathrm{OBS}_1, \ldots, \mathrm{OBS}_k$.If the algorithm returns a Vertex Set of size $\le k$, and we verify it, then we know $G \in \mathrm{VC}_k$.*

Algorithm 6 *We have* an algorithm that:
On input i, output H_i. Don't care about time.