BILL, RECORD LECTURE!!!!

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Computable Ramsey Theorem For Graphs

Exposition by William Gasarch

March 26, 2026

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I will not prove this, but I will use it.

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We give the first few steps of the process.

Examples of The First Few Steps of The Process

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Look at $x_1 = 1$ and all of the edges coming out of it:

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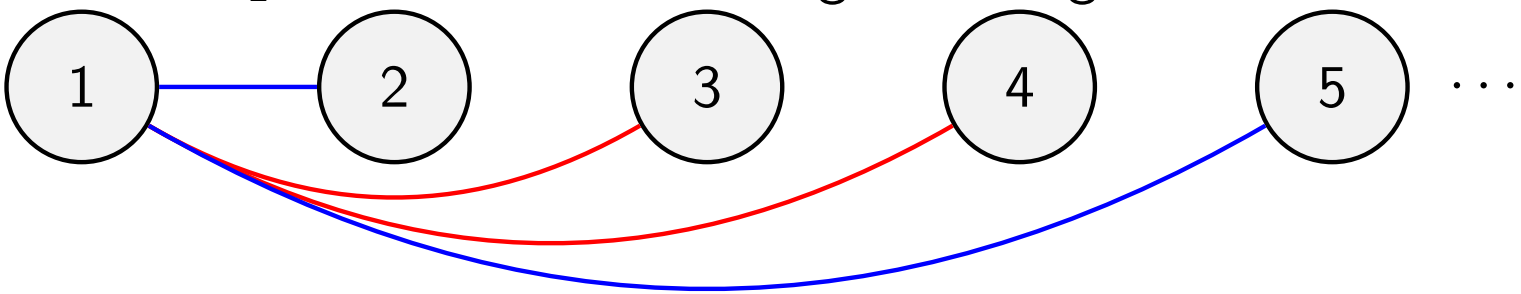

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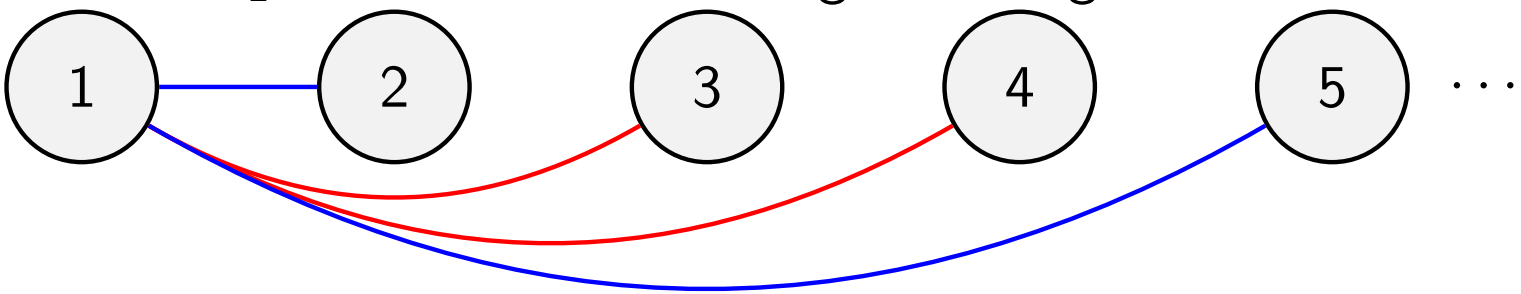

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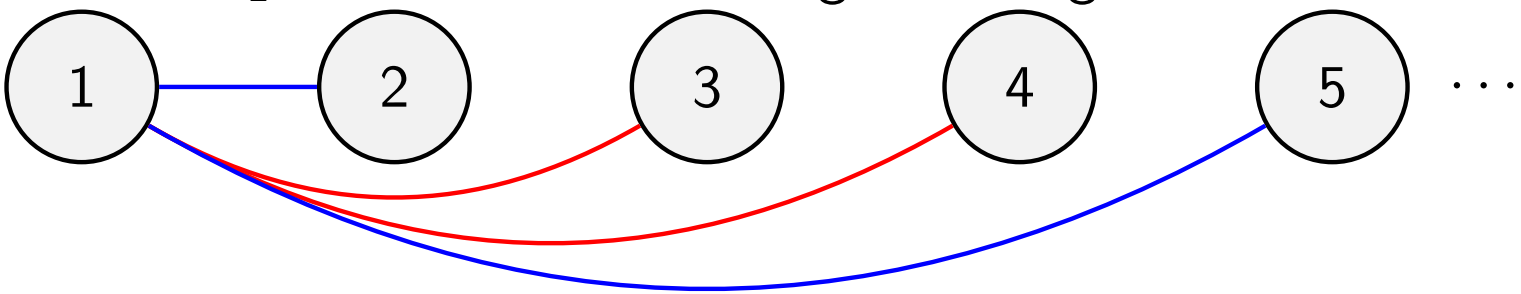

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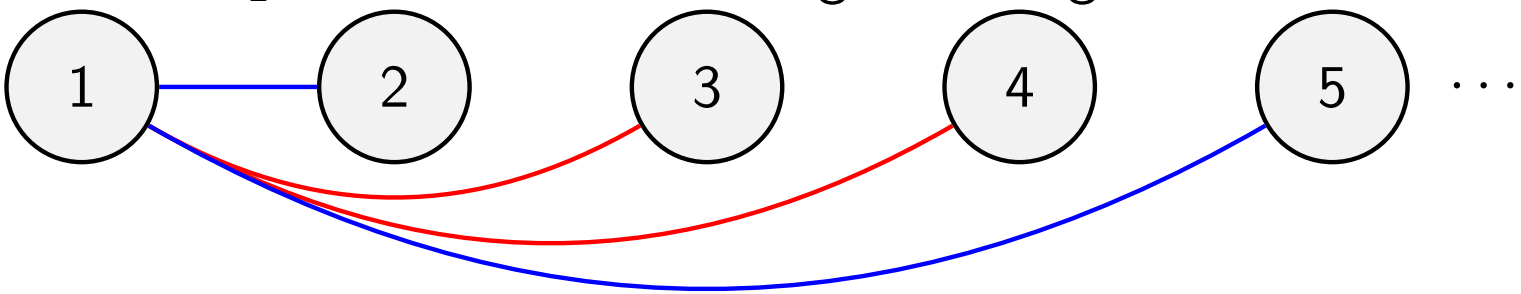

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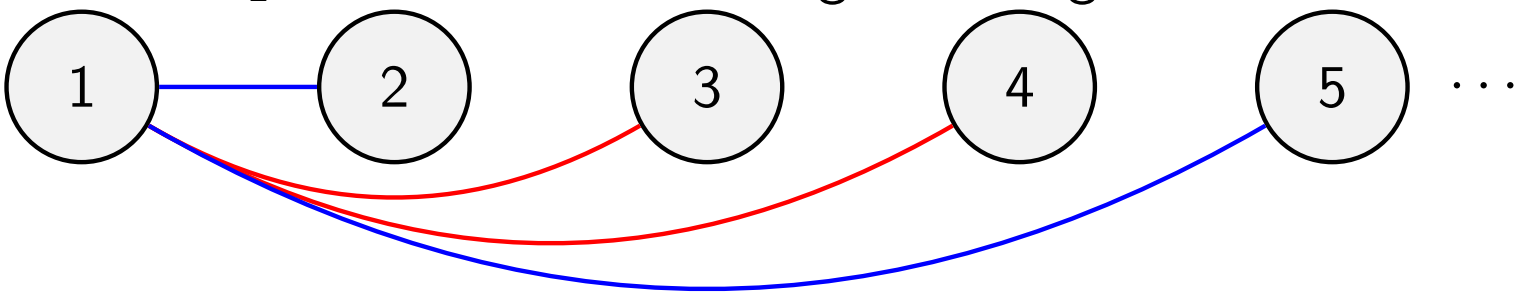

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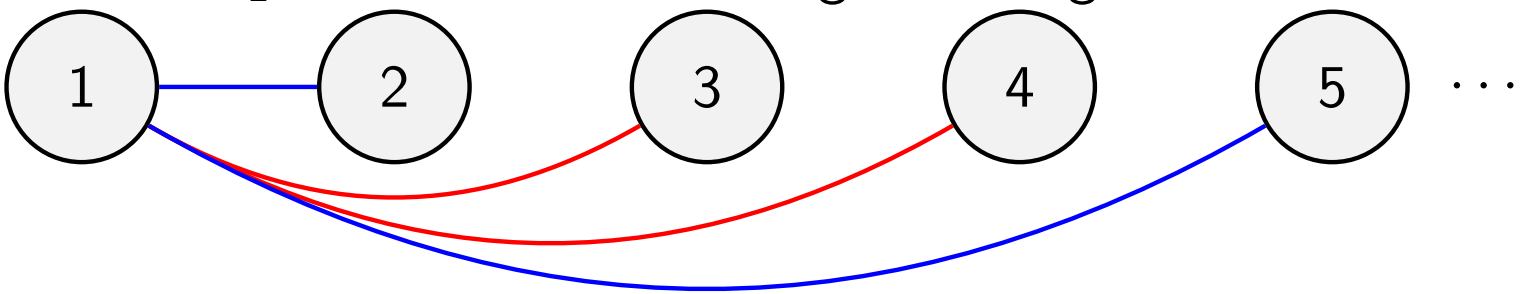

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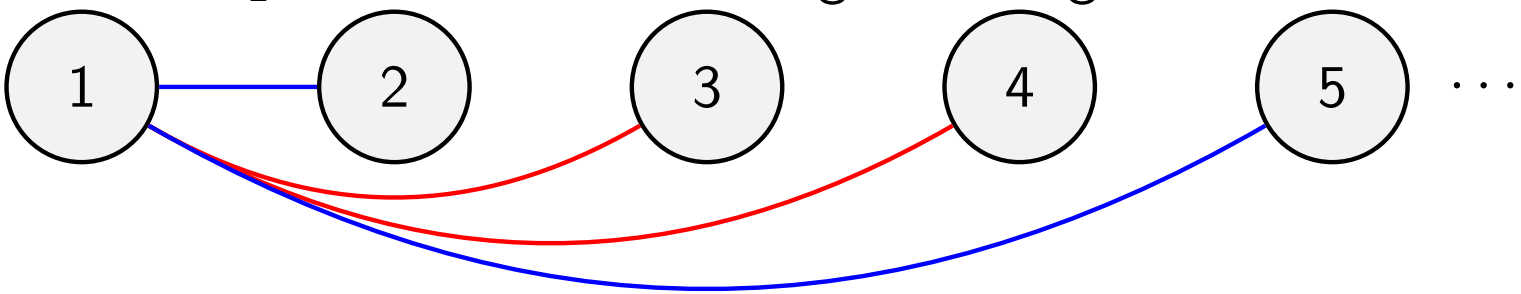

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We have a picture of this on the next slide.

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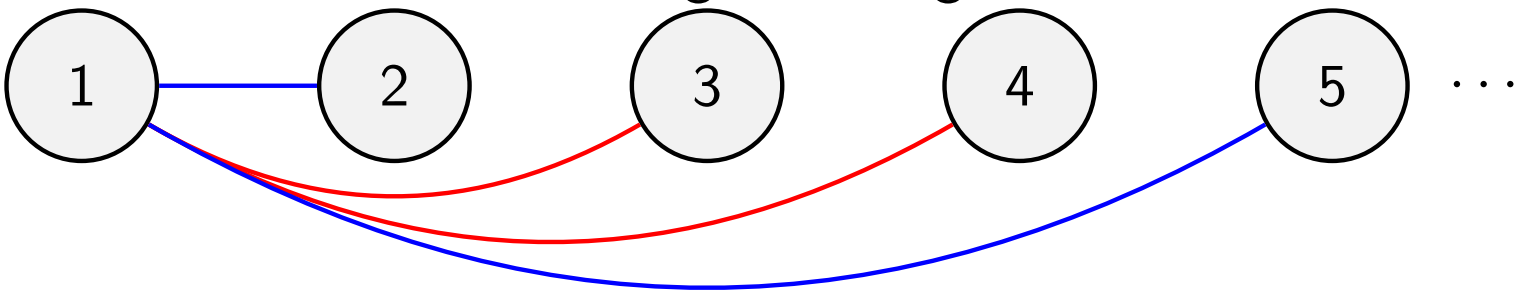

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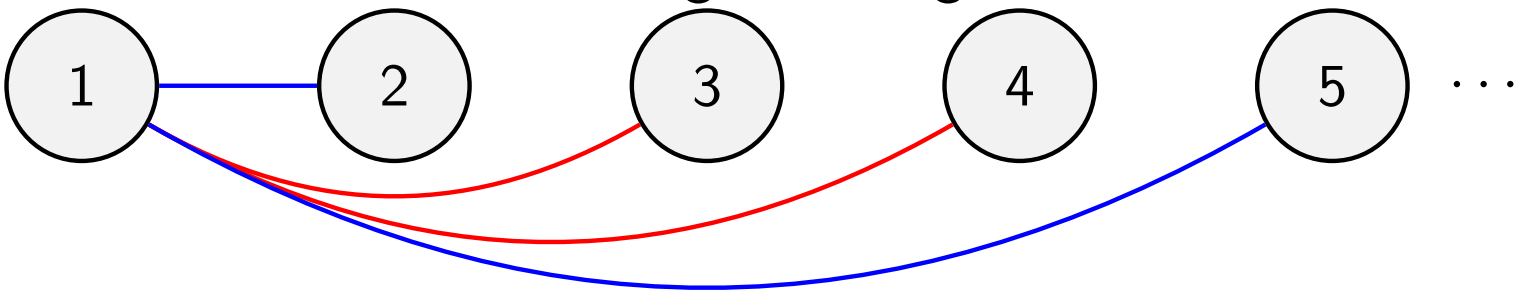

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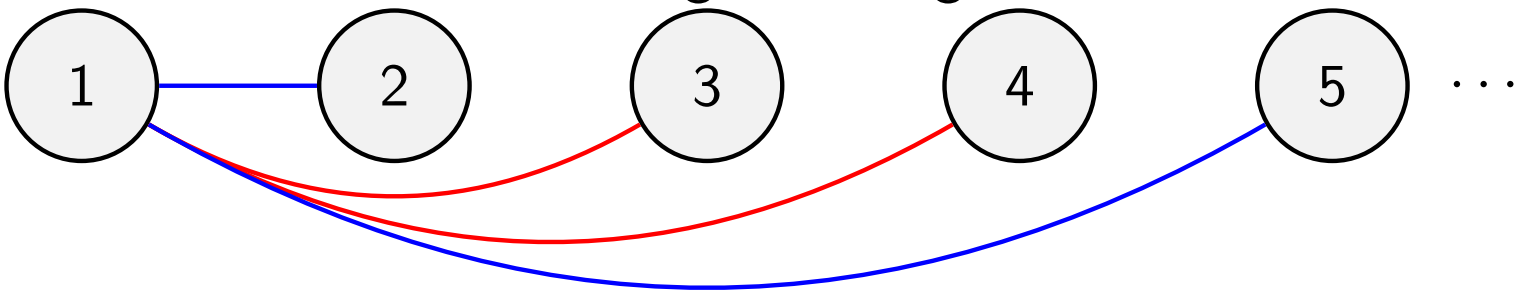

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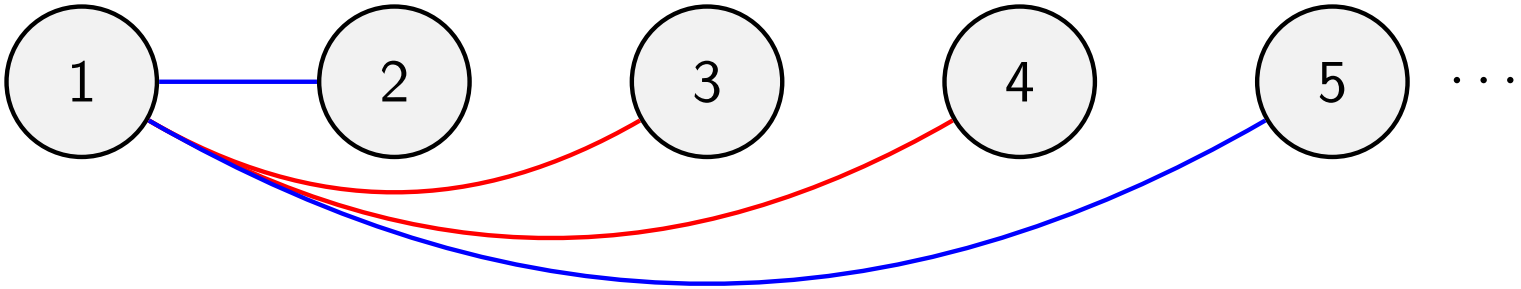

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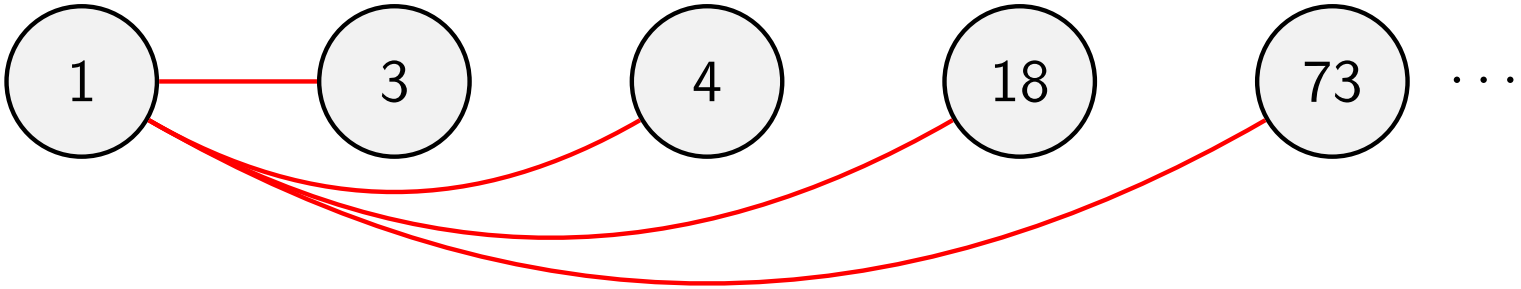

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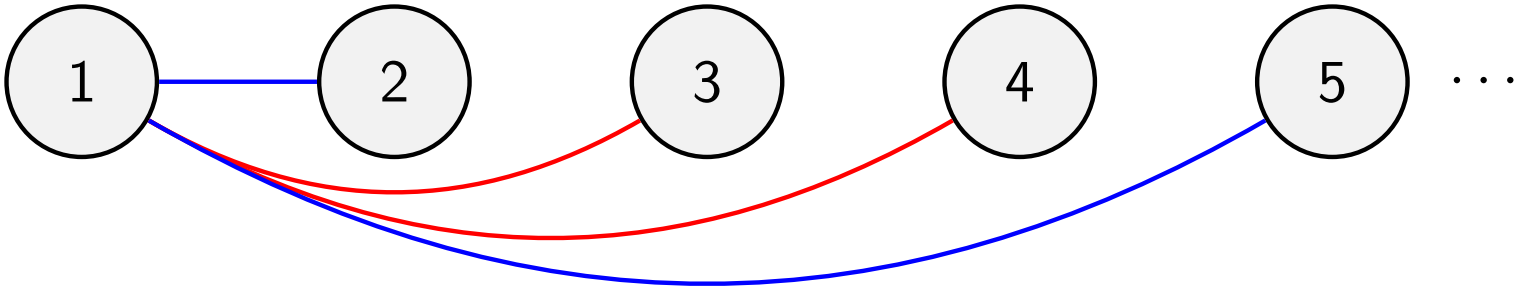

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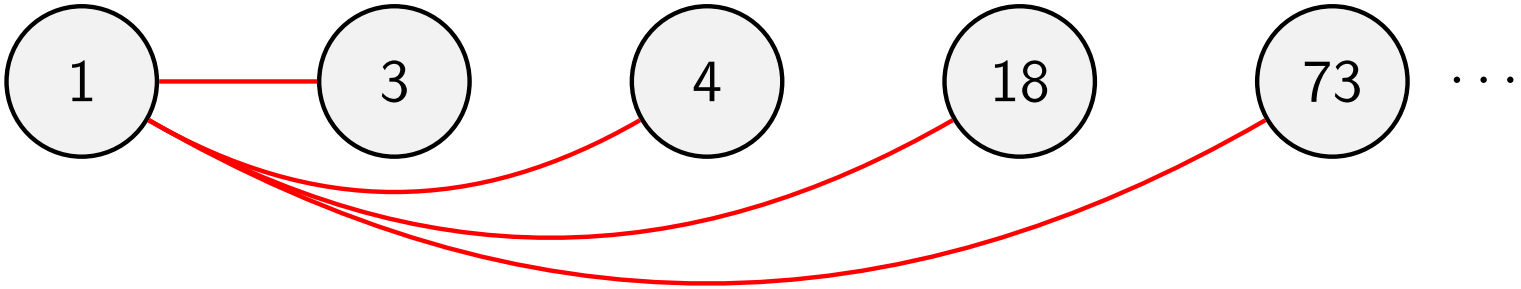

We Omit 1 from future pictures but its **Still Alive and Well.**
`https://www.youtube.com/watch?v=8--jVqaU-G8`.

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What about c_2. This is the **Key**.

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Does this limit the applicability of the theorem?

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If there are an ∞ number of **B** points then **ALG-BLUE** shows there is an ∞ Σ_3- **B**-homog set

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The proof is nonconstructive.

It cannot be made constructive. I probably proved that.

Does this limit the applicability of the theorem?
Not even a little!

Better/More Is Known for Comp. Inf Ramsey

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Open Narrow the gap between Π_{2a-1} and Σ_a.