

BILL, RECORD LECTURE!!!!

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Project Re-derive the known obs sets with a SAT Solver and (try to) find the obs set for 5-coloring.

Ramsey Games

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Look at avoidance game- try to NOT get K_3 .

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Caveat I've had some HS students do this before for Graphs. Might want to do it for VDW's Theorem, Poly VDW, Square-Thm, Grid-Thms.

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Combinations of the above. Empirical studies of the above.

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Projects Good writeup of the known proof(s) of this, perhaps better bound on the numbers. Possibly empirical results.

Coloring the Plane with a Ctable number of colors

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What happens if you color \mathbb{R}^2 with a countable number of colors?

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Project Read, understand, and write up these results. Then see if you can extend.

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Darling's Problem: If you color \mathbb{R}^3 what happens?

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For all of the above, more colors.

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Survey of L_a - L_b results- build on what Kelin and Chaewoon have done.

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smallskip In \mathbb{R}^3 try to get mono tetrahedron.

Anti-Ramsey

Survey of the papers I have on Anti-Ramsey.

The papers I wrote up (with co-author) seem tedious- automate?