BILL, RECORD LECTURE!!!!

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Ramsey Theory For $\binom{\mathbb{R}}{2}$

Exposition by William Gasarch

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We will see later that its a problem for **me!**

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$\mathbb{R}$ can be well ordered. Is that strange?

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$(\forall x \in \mathbb{R})(\exists x^+)$ such that $x \prec x^+$ and $\neg\exists y[x \prec y \prec x^+]$.

Odd? Do these two odd facts make your doubt WOP?

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Note: $\mathbb{R} = \bigcup_{i=1}^{\infty} A_i$.

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We show that any homog set is countable.

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We show $\forall i$, A_i is ctble, so H is ctble.

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First Key

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Note If $x \in H$ then $x^{+!} \in H$ then **both** $x \prec x^{+!}$ and $x < x^{+!}$.

We show $|A_{1/i} \cap (\frac{j}{i}, \frac{j+1}{i})| \le 1$. We do an example.

$A_{0.1} = \{x \colon |x^{+!} - x| > 0.1\} = \{x \colon x^{+!} - x > 0.1\}$

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Generally, $(\forall i, j)[|A_{1/i} \cap (\frac{j}{i}, \frac{j+1}{i})| \le 1]$.
Hence

$$H = \bigcup_{i=1}^{\infty} \bigcup_{j=1}^{\infty} A_{1/i} \cap \left(\frac{j}{n}, \frac{j+1}{n}\right) \text{ is countable.}$$

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Can we make Ramsey over the reals true?

The Problems With Set Theory

What are The Right Axioms

Here at WCOZ · · ·

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The Disc Jockey Suggested Examine, ZFC, the usual axiom system, carefully.

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They will be on the next few slides.

Do We Even Know If ZFC Is Consistent?

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People are less worried now since no contradiction has been found from 1909 until now.

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What to make of that?

Cohen's View of CH ind of ZFC

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Next slide for an axiom candidate.

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Argument For Inac. Cards Odd if $\mathbb{N}$ was the **only** such cardinal.

What About Ramsey's Theorem For Sets Bigger Than $\mathbb{R}$?

Ramsey Cardinals (this is not quite standard but fine for us)

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Answer on next slide.

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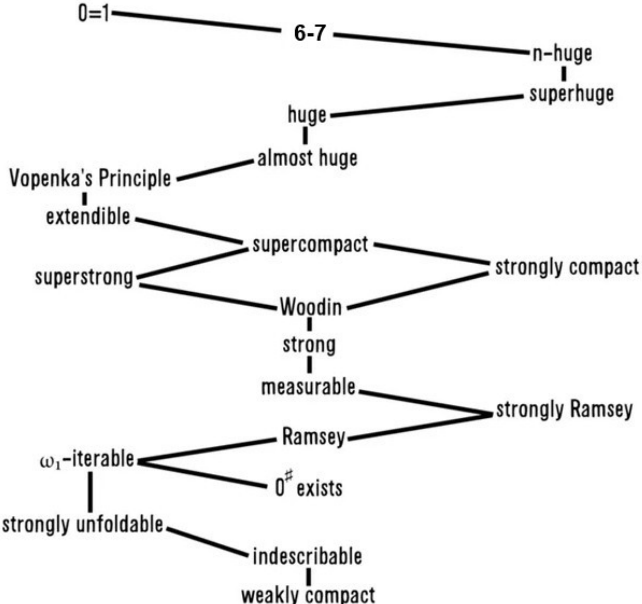
There are many.
Even Within Ramsey There are Many RC , Ineffably RC, Almost RC, Strongly RC, and more .

Large Cardinals



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Now most set theorists think CH has no fixed value but are curious what happens when you assume CH an also when you assume NOT(CH).

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The Axiom of Choice Seems True But Darling Thinks Its False

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Except when its really important, as in the next few slides.

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An Application of The BT Paradox

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BILL: Let me guess the problem: There are some miracles that cannot fit into modern science.

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LEO: Exactly! And I hope that Christians who are scientists (not to be confused with Christian Science (see here)) will take up the study of miracles and see how they can fit into modern science.

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