BILL, RECORD LECTURE!!!!

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Well Quasi Order And Subsequences

Exposition by William Gasarch-U of MD

Our Motivating Question

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$L \subseteq \{a, b\}^*$ is often called **a language**.

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Example

$$\mathrm{SUBSEQ}(aaba) = \{e, a, b, aa, ab, ba, aaa, aab, aba, aaba\}.$$

If L is Regular then $\mathrm{SUBSEQ}(L)$ is Regular

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L is regular $\implies$ $\mathrm{SUBSEQ}(L)$ is regular.

Take the DFA for L and put between any two states add an e-transition.

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The proof uses that if there is an NFA for L then there is a DFA for L.

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The idea is clear.

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Perhaps if we assume $\mathrm{P} \neq \mathrm{NP}$?

Vote on P

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1) Known: **if** $L \in \mathrm{P}$ **then** $\mathrm{SUBSEQ}(L) \in \mathrm{P}$**.**
2) Known: $\exists\ L \in \mathrm{P}$ **with** $\mathrm{SUBSEQ}(L) \notin \mathrm{P}$**.**
3) Known: **If** $\mathrm{P} \neq \mathrm{NP}$ **then** $\exists\ L \in \mathrm{P}$ **with** $\mathrm{SUBSEQ}(L) \notin \mathrm{P}$**.**
4) None of the above are known.

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Thm Let $(X, \preceq)$ be a wqo. Let $Y \subseteq X$ be closed downward. Then there exists a **Finite Obstruction Set** for Y.
(This is an easy generalization of the Theorem about a set of graphs closed downward under minor has a finite obs set.)

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If $y \in \mathrm{SUBSEQ}(L)$ then there exists $z \in L$, $y \preceq z$.

If $w \preceq y$ then $w \preceq y \preceq z$ so $w \preceq z$.

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Hence $\exists$ a finite obstruction set $z_1, \ldots, z_L$ for $\mathrm{SUBSEQ}(L)$.

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See next page for exciting finish!

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BEGIN ALG for $\mathbf{SUBSEQ}(\boldsymbol{L})$

Input(y)

For $i = 1$ to L test if $z_i \preceq y$

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For $i = 1$ to L test if $z_i \preceq y$

If $(\forall i)[z_i \not\preceq y]$ then output **YES**

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How hard is it to determine if $z \preceq y$? Discuss

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Thm If $L \in \mathrm{P}$ then $\mathrm{SUBSEQ}(L)$ is regular.

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Thm If L is any subset of Σ^* then $\mathrm{SUBSEQ}(L)$ is regular.

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No such statement is true for P or higher classes.

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You can guess the next sentence
I proved it had to be nonconstructive.

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The question

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$$\textbf{Does } L \in \mathrm{P} \textbf{ imply } \mathrm{SUBSEQ}(L) \in \mathrm{P}\textbf{?}$$

sounds interesting.

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The question

Does $L \in \mathrm{P}$ **imply** $\mathrm{SUBSEQ}(L) \in \mathrm{P}$**?**

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The answer

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The question

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If L is any set then $\mathrm{SUBSEQ}(L)$ is regular

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Its only in his book because I told it to him.

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So $\mathrm{SUBSEQ}(L)$ has very little to do with L.

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Why Not They were math people while **I walk the line** (Thats a song I wrote and sold to Johnny cash: `https://www.youtube.com/watch?v=jh169rVMveA`.)

Who First Proved The All L Theorem (not a joke)

Who first proved

If L is any set then $\mathrm{SUBSEQ}(L)$ is regular

Hard to say.

People in math had proven that If X is a wqo then $\mathrm{SUBSEQ}(X^*)$ is a wqo.

People in math had proven that if Y is closed downward then it has a finite obstruction set.

But nobody had put these two together until I did.

Why Not They were math people while **I walk the line**
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Moral of the Story If you have knowledge of two **different** fields then you can do things nobody else can do.