# The Yao Cell Probe Model

# The Model

**Definition:** An $f(n,u)$ *probe Data Structure for Membership* consists of two things:

- A function $PUT : \binom{[u]}{n} \rightarrow S_n$. We think of this as putting the $n$ elements into an array of length $n$. Call this array $A$
- A non-adaptive algorithm that will, given $x \in [u]$, make $\leq f(u,n)$ probes to the array $A[1 \ldots n]$ and outputs YES if $x$ is in the array, and NO if not.

# Examples

**Standard Example:** Store the items in sorted order and use binary search. In this case $f(n, u) = \lceil \lg_2(n) \rceil$. Works for any $n \leq u$.

**Stupid Example:** $u = n$. Put the element in in sorted order (doesn't matter). Always answer YES. $f(n, u) = 0$.

**Slightly Less Stupid Example:** $u = n + 1$. There is exactly one element, $x$, NOT in array. Put $x + 1 \pmod{u}$ into $A[1]$. Put other elements in sorted (doesn't matter). One query to $A[1]$ tells you what all MEM question.

NOTE- in last example, answer MEM question, but NOT where in the table it is. That's okay!

# How well can you do with One Probe?

DO IN CLASS:

1. Can you do 1-probe if $u = n + 2$? $u = n + 3$?
2. Find some function $q$ such that if $u = q(n)$ then you CANNOT do in 1-probe.

# When is Log n Required?-A Needed Lemma

We find a function $q(n)$ such that if $u \geq q(n)$ then REQUIRES $\lceil \lg n \rceil$ probes.
Need Lemma. Leave proof to you. Uses induction and Adversary arg.
**Lemma:** Assume $u \geq 2n - 1$.

1. If the *PUT* function always outputs INCREASING (so elts are put in table in inc order) any probe algorithm must take $\geq \lceil \lg(n) \rceil$.
2. If the *PUT* function is CONSTANT then any probe algorithm must take $\geq \lceil \lg(n) \rceil$.

# When is Log n Required?

**Theorem:** There is a function $q(n)$ (TBD) such that if $u \geq q(n)$ then $\lceil \lg n \rceil$ probes are REQUIRED.

**Proof**

Take the function PUT. From it, create the following coloring:
$COL : \binom{[u]}{n} \rightarrow [n!]$: Map $A$ to the perm its mapped to.
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RAMSEY"S THEOREM!
What parameters: $n$-ary, $n!$ colors, homog set of size $2n-1$. So need $u \geq R_n^{n!}(2n-1)$.

Let $H$ be that homog set. PUT restricted to $\binom{H}{n}$ is constant so by lemma takes $\lceil \lg n \rceil$ probes.