Take Home Final. Given out Apr 23
Morally Due TUESDAY May 8. Sick Cat Day-Thursday May 10
THIS EXAM IS THREE PAGES!!!!!!!!!!!!!!!

1. (0 points) What is your name? Write it clearly. Staple this.

2. (20 points)

   (a) Prove the following. There exists a function f such that the following holds:

   If $T_1, T_2, \ldots, T_{f(k)}$ is a FINITE sequence of trees, where T_i has at most $k(i+1)$ nodes, there is an uptick.

   For this problem the trees are ordered as $T_1 \le T_2$ if T_1 is a minor of T_2.

   (b) (Was there anything special about $k(i+1)$?) Is there some function $g(i,k)$ such that if you replace the $k(i+1)$ in the first question with $g(i,k)$. the theorem is now false?

   (c) (Think About) Back to the first question: can you find bounds on $f(k)$?

3. (20 points) Find a function $f(c,k)$ such that the following is true, and prove it:

   There exists a c-coloring of $[f(c,k)] \times [f(c,k)]$ with no monochromatic $k \times k$ regular grids.

   A regular grid is a square grid with the points equally spaced.

   Note: for full credit you will have to provide a decent bound; answering $f(c,k) = 0$ is technically correct but will receive no points.

GO TO THE NEXT PAGE

4. (20 points) (You may use Ramsey's theorem and the known bounds on it from this class, in this problem.)

   (a) Show that the following holds:
   Bill gives you a 2-coloring of N AND $\binom{\mathsf{N}}{2}$. Show there exists an infinite $X \subseteq \mathsf{N}$ such that it is BOTH 1-homog (so all the vertices are the same color) and 2-homog (all of the pairs are the same color). It need NOT be the case that these two colors are the same.

   (b) find a function $f(k)$ such that the following holds:
   Bill gives you a 2-coloring of $[f(k)]$ AND $\binom{[f(k)]}{2}$. Show there exists a set $X \subseteq [f(k)]$ of size k such that it is BOTH 1-homog (so all the vertices are the same color) and 2-homog (all of the pairs are the same color). It need NOT be the case that these two colors are the same.

5. (20 points) Find a function $f(c)$ such that the following is true, and prove it:

   For all $c \geq 2$,

   (a) There is a c-coloring of the $[c+1] \times [f(c)-1]$ with NO monochromatic rectangles. (There are no four points that form the corners of a rectangle that are the same color.)

   (b) Every c-coloring of the $[c+1] \times [f(c)]$ has a monochromatic rectangle.

GOTO THE NEXT PAGE!!!!!!!!!!!!!!!!

6. (20 points)

   Definition: Let e and $f_1 < \cdots < f_n$ are all in N. Then

$$METZ(e; f_1, \ldots, f_n) = \{d+f_1b_1+f_2b_2+\cdots+f_nb_n : b_1, \ldots, b_n \in \{0,1\}\}$$

   We call such a set an n-METZ.

   Example:

$$METZ(5; 1, 3, 4) = \{5, 5+1, 5+3, 5+4, 5+1+3, 5+1+4, 5+3+4, 5+1+3+4\}$$

$$= \{5, 6, 8, 9, 10, 12, 13\}$$

   (It might not have 2^n elements since some sums are the same.)

   Let $H(n, c)$ be the least H such that for all c-colorings of $[H(n, c)]$ there exists a monochromatic n-METZ. In this problem you will show, in two ways, that $H(n, c)$ exists.

   (a) Show USING VDW's theorem that, for all c, n, there exists $H = H(n, c)$ such that every c-coloring of $[H]$ has a monochromatic n-METZ. In particular bound $H(n, c)$ using VDW numbers.

   (b) Show that for all c, n, there exists $H = H(n, c)$ such that every c-coloring of $[H]$ has a monochromatic n-METZ WITHOUT using VDW's theorem. More precisely- find a recurrence that enables one to compute a bound for $H(m, c)$ that does not involve VDW numbers.

7. (0 points- but you MUST ANSWER THIS. If you do not then I will DEDUCT 10 points) Name two things that could be done to improve the course. If you have more than two thats fine. They should be either constructive (so I really can use them next time I teach) or funny (so Erik and I can have a laugh while grading).