Homework 2, Morally Due Tue Feb 13, 2018

COURSE WEBSITE: http://www.cs.umd.edu/˜gasarch/858/S18.html
(The symbol before gasarch is a tilde.)

HOMEWORK IS TWO PAGES!!!!!!!!!!!!!

1. (0 points) What is your name? Write it clearly. Staple your HW. When is the midterm tentatively scheduled (give Date and Time)? If you cannot make it in that day/time see me ASAP.

2. (40 points) The infinite Ramsey Theorems we have proven only cared about the CARDINALITY of the homog set (it had to be infinite) and not how it was ordered. In this problem we will look at coloring $\binom{\mathsf{Z}}{a}$ and and $\binom{\mathsf{Q}}{a}$ and see if we can get a homog set with a proscribed order type.

   Definition: A set has H has *order type* Z if there is an order preserving bijection from H to Z. A set has H has *order type* Q if there is an order preserving bijection from H to Q. You can replace Z and Q with any linear orderings.

   (a) (13 points) Prove of Disprove: For all 2-colorings of $\binom{\mathsf{Z}}{1}$ there exists a homog set of order type Z. (In more normal words: for all 2-colorings of Z there is a monochromatic subset that is infinite in both directions.)

   (b) (13 points) Prove of Disprove: For all 2-colorings of $\binom{\mathsf{Q}}{1}$ there exists a homog set of order type Q. (You may use that a countable dense set with no endpoints is of order the Q.)

   (c) (14 points) Prove of Disprove: For all 2-colorings of $\binom{\mathsf{Z}}{2}$ there exists a homog set of order type Z.

   (d) Challenge Question (Bill and Erik thought this was hard until Erik solved it. Now they think it's easy.) Prove of Disprove: For all 2-colorings of $\binom{\mathsf{Q}}{2}$ there exists a homog set of order type Q. (You may use that a countable dense set with no endpoints is of order the Q.)

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3. (60 points)

Recall the proof of the canonical Ramsey theorem using a 16-coloring and the 4-ary Ramsey theorem

(https://www.cs.umd.edu/users/gasarch/COURSES/858/S18/infcanramseytalk.pdf).

(a) (15 points) Show that given an infinite homogenous set H of color 4 (under the 16-coloring), there is either an infinite homogenous set or an infinite min-homogenous set (under the original coloring). Recall that $COL'(x_1 < x_2 < x_3 < x_4) = 4$ if $COL(x_2, x_3) = COL(x_2, x_4)$.

(b) (15 points) Show that given an infinite homogenous set H of color 6 (under the 16-coloring), there is either an infinite homogenous set or an infinite max-homogenous set (under the original coloring). Recall that $COL'(x_1 < x_2 < x_3 < x_4) = 6$ if $COL(x_1, x_4) = COL(x_2, x_4)$.

(c) (15 points) Show that given an infinite homogenous set H of color 11 (under the 16-coloring), there is an infinite homogenous, min-homogenous, or max-homogenous set (under the original coloring). Recall that $COL'(x_1 < x_2 < x_3 < x_4) = 11$ if $COL(x_1, x_2) = COL(x_3, x_4)$.

(d) (15 points) Show that given an infinite homogenous set H of color 12 (under the 16-coloring), there is an infinite homogenous set (under the original coloring). Recall that $COL'(x_1 < x_2 < x_3 < x_4) = 12$ if $COL(x_1, x_3) = COL(x_2, x_4)$.