Homework 4, Morally Due Tue Feb 20, 2018

COURSE WEBSITE: `http://www.cs.umd.edu/~gasarch/858/S18.html`

1. (0 points) What is your name? Write it clearly. Staple your HW. What type of midterm will there be?

2. (25 points) In class we showed that for all n, $R(n)$ exists. The proof DID NOT give any bounds on $R(n)$. Use a similar proof for the following:

   Let $R(a, c, n)$ be such that for all c-colorings of $\binom{[R(a,c,n)]}{a}$ there exists a homogenous set of size n. Show that $R(a, c, n)$ exists.

3. (25 points) In class we showed that for all n, $LR(n)$ exists. The proof DID NOT give any bounds on $R(n)$. Use a similar proof for the following:

   Let $LR(a, c, n)$ be such that for all c-colorings of $\binom{\{n,n+1,\ldots,LR(a,c,n)\}}{a}$ there exists a large homogenous set.

   Show that $LR(a, c, n)$ exists.

4. (25 points) Prove the following using some Can Ramsey Theorem: (Countable means infinite - some books disagree but they are wrong.)

   If $X \subseteq \mathsf{R}^3$ is a countable set of points, no four on the same plane, there exists countable $Y \subseteq X$ such that every 4-subset of Y yields a different volume.

5. (25 points) (For his problem assume that there is NO cardinality between countable and the cardinality of the reals.) We say $|X| = |\mathsf{R}|$ to mean that X and R are the same size, so there is a bijection between them.

   Prove the following using a Maximal Set argument:

   If $X \subseteq \mathsf{R}^3$, $|X| = |\mathsf{R}|$, no four on the same plane, there exists $Y \subseteq \mathsf{R}^3$, $|Y| = |\mathsf{R}|$, such that every 4-subset of Y yields a different volume.

6. (0 points but you must do this so we can discuss) On the course website is a link to a review of a book on the Banach-Tarski Pardox. Read the review. Be prepared to discuss if you think the BT paradox is TRUE or FALSE or SOMETHING ELSE. There is no right answer here but I want to know what you think.