

Homework 5, Morally Due Tue Mar 27, 2018
NOTE - THIS HW IS TWO PAGES LONG

1. (0 points) What is your name? Write it clearly. Staple your HW.
2. (30 points) Let $a \in \mathbb{N}$, Let $c \in \mathbb{N}$. The language of c -colored a -hypergraphs will have just $E_i(x_1, \dots, x_a)$ for $1 \leq i \leq c$.

Let ϕ be a sentence in the language of c -colored a -hypergraphs of the form

$$(\exists x_1) \cdots (\exists x_m)(\forall y_1, \dots, y_L)[\psi(\vec{x}, \vec{y})].$$

Show that

- (a) The spec of ϕ is either finite or cofinite.
 - (b) The function that, given any ϕ as above, outputs the spec, is computable.
3. (40 points) Let (W, \leq) be a wqo. Let $TREEW$ be the set of trees where the nodes are labeled with elements of W . We define $T \preceq T'$ if you can remove vertices, remove edges, contract edges, until you get a tree T'' such that the vertices of T are \leq their analogs in T' .

Show that $TREEW$ under \preceq is a wqo (you already did one of the main steps on the take home midterm — if W is a wqo then the set of all finite subsets of W is a wqo).

4. (30 points) The $n \times m$ grid is the set of points

$$\{(a, b) : 1 \leq a \leq n \text{ and } 1 \leq b \leq m\}.$$

In this problem we will be coloring these points.

A *monochromatic rectangle* is when there are FOUR points that are the corners of a rectangle that are all the same color. Example would be

$$\{(3, 4), (3, 8), (7, 4), (7, 8)\}.$$

Find EXACTLY which grids CAN be 2-colored without having a monochromatic rectangle.

THERE IS ANOTHER PAGE TO THIS HW

5. (Extra Credit (so to impress me for a letter or some such)) Find EXACTLY which grids CAN be 3-colored without having a monochromatic rectangle.
6. On the course website is (1) Gangsta Paradise (2) Mathematics Paradise and the lyrics, by the Klein Four (listen to it while reading the lyrics) (3) A different Mathematics Paradise song, (4) Amish Paradise by Weird Al

Listen to all four (reading the lyrics at the same time for (2)). For each one rate them either: Awesome, Very Good, Good, Uh- Okay I guess, So Bad its good, Just Bad, Ears bleeding.