Homework 7, Morally Due Tue Apr 10, 2018
THIS HW IS TWO PAGES

1. (30 points) Let $R(a, b, c)$ be the least n such that for all 3-colorings of $\binom{[n]}{2}$ there exists either a RED K_a or a BLUE K_b or a GREEN K_c.

   (a) State and prove theorems about $R(a, b, c)$ (see next question so you'll now what you are aiming for).

   (b) Use the theorems you proved in the first part to obtain by hand bounds on $R(3, 3, 3)$.

   (c) Use the theorems and a computer program to obtain bounds on $R(a, b, c)$ for $1 \le a \le b \le c \le 10$. Submit pseudocode and your results.

2. (30 points) Write programs for the following.

   (a) Input: A colored K_n and a number k. Output: YES if there is NO homog set of size k, NO if there is one.

   (b) Input: n. On this input you produce 100 RANDOM colorings of K_n by, for each edge, coloring it RED or BLUE with equal probability. For each coloring note the first k such that there is NO homog set of size k. Over the 100 RANDOM colorings, output the min k.

   (c) For $n = 10$ to 20 use the above procedures to find graphs that do not have large homog sets. For each n report the smallest k found such that there is 2-colored graph with no homog set of size k.

   Submit your pseudocode and your results.

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3. (40 points) (In this problem we do a 3-d version of the Klein-Erdos-Szekeres theorem.)

   First some definitions:

   - A set of points in R^3 is *in general position* (or GP) if no 4 points are on the same plane. (This is analogous to no-3-points-colinear.)
   - A set of 4 points Y in R^3 that are in GP enclose a shape which we call a 4-gon. We denote this by $4gon(Y)$. (This is analogous to triangles.)
   - A GP set of points X in R^3 is *metz* if for all $Y \in \binom{X}{4}$ there is no point of X inside $4gon(Y)$. (This is analogous to a convex k-gone.) A set like this of size k we call a k-metz set.

   AND NOW the question:

   Show that for all k there exists n so that for any GP set of n points in R^3 there exists a k- metz set.

4. (0 points but do it) This week two math songs from the TV show *Square One TV*, a show on Public TV that taught kids some math. I enjoyed it very much when I watched it ... at age 40.

   - 8% of my love
   - Thats Combinatorics!