

Homework 10, Morally Due Tue May 1, 2018

1. (0 points) Give your name! What kind of final will we have? When is it due?
2. (30 points) Give a 2-coloring of \mathbb{N} that does NOT have an INFINITE monochromatic Arithmetic progression and prove that it has no such progression.

SOLUTION TO PROBLEM TWO

For n even color $\{2^n, \dots, 2^{n+1} - 1\}$ RED

For n odd color $\{2^n, \dots, 2^{n+1} - 1\}$ BLUE.

So

0,1 RED

2,3, BLUE

4,5,6,7 RED

8,9,10,11,12,13,14,15 BLUE

Assume, by contradiction, that $a, a + d, a + 2d, \dots$ is a mono ω -AP.

Let i be such that $2^i \leq d \leq 2^{i+1} - 1$.

Look at the sequence in the block of size 2^{i+1} . Let $a + nd$ be the largest element of the sequence in that block. The next number of the same color is $> 2^{i+1}$ away and hence $a + (n + 1)d$ cannot be the same color as $a + nd$.

END OF SOLUTION TO PROBLEM TWO

3. (30 points) Show that for all k, c, s there exists $J = J(k, c, s)$ such that, for all c -colorings of $[J]$, there exists a, d such that the following are all the same color:

$$a, a + d, a + 2d, \dots, a + (k - 1)d, sd$$

SOLUTION TO PROBLEM THREE

Omitted

END OF SOLUTION TO PROBLEM THREE

4. (40 points) A tuple of integers (a_1, \dots, a_n) is called *awesome* if the following hold:

For all c , for all c -colorings of \mathbf{N} , there exists monochromatic (e_1, \dots, e_n) such that $\sum_{i=1}^n a_i e_i = 0$ (note- the e_1, \dots, e_n can have repeated elements, and for this problem 0 is not a natural number).

- (a) Show that if SOME subset of the a_i 's sum to 0, then (a_1, \dots, a_n) is awesome.
- (b) Show that if NO subset of the a_i 's sum to 0, then (a_1, \dots, a_n) is NOT awesome.

SOLUTION TO PROBLEM FOUR

Omitted- will do in class.

END OF SOLUTION TO PROBLEM FOUR

5. (0 points but you'll want to do this!) Who was the first person to write novelty songs about math. It may have been Tom Lehrer. I have collected on my website four math-songs and two chemistry songs that he wrote. Listen! Enjoy! In class we'll discuss what you liked and didn't like. Even if you know of his work, some of the songs here are obscure in that they are not on any of his albums. On the other hand, how can something be obscure if its on youtube?