Homework 11, Morally Due Tue May 8, 2018
THIS HW IS TWO PAGES!!!!!!!!!!!!!!!!!!!

1. (0 points) Give your name! What kind of final will we have? When is it due?

2. (40 points) In this problem you may assume that, for all k, for all c, there exists $N = N(k,c)$ such that for all c-colorings of $[N] \times [N]$ there exists a monochromatic regular $k \times k$ grid.

   Show that there exists M such that, for all 2-colorings of $[M] \times [M] \times [M]$, there exists four points that are the same color of the following form:

   (x, x_2, x_3)

   $(x + \lambda, x_2, x_3)$

   $(x, x_2 + \lambda, z)$

   $(x, x_2, x_3 + \lambda)$

   (Such a set of four points is called a 4-Ajeet.)

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3. (60 points) Consider the following problem in communication complexity: Let $k \geq 3$ be a constant and n is large. k people each have an n-bit number on their forehead. We denote these numbers $x_1, \ldots, x_k$. They want to know if

$$x_1 + \cdots + x_k = 2^n$$

In class we showed that if $k = 3$ they can do this in $O(n^{1/2})$ bits of communication. Find a sequence $\alpha_3 \geq \alpha_4 \geq \alpha_5 \geq \cdots$ such that

- k people can do this problem with $O(n^{\alpha_k})$ bits of communication.
- $\lim_{k\to\infty} \alpha_k = 0$

(Hint1: Follow the proof of the same problem for 3 people trying to determine $x_1 + x_2 + x_3 = 2^n$.

Hint2: You will need k-free sets. Look at the course website for a paper on k-free sets. Use the main theorem of this paper.

Advice: Use the theorem that turns k-free sets into colorings $[n]$ (later $[kn]$) that have no mono k-AP's.

Terminology to make Erik's grading easier: A k-*Ajeet* is a set of k points in N^{k-1} of the following form:

$(x_1, \ldots, x_{k-1})$

$(x_1 + \lambda, x_2, \ldots, x_{k-1})$

$(x_1, x_2 + \lambda, x_3, \ldots, x_{k-1})$

$\vdots$

$(x_1, x_2, x_3, \ldots, x_{k-1}, x_{k-1} + \lambda)$