Take Home Final
Due May 20 at 7:00PM. No Extensions
THIS EXAM IS THREE PAGES!!!!!!!!!!!!!!!

1. (0 points) What is your name? Write it clearly.

2. (20 points) A coloring $COL : \binom{\mathsf{N}}{2} \rightarrow \mathsf{N}$ is *Dhruv* if, for all $x < y$,

$$x \le COL(x, y) \le y.$$

   (a) Either give an example of a Dhruv coloring $COL : \binom{\mathsf{N}}{2} \rightarrow \mathsf{N}$ that has an infinite homog set OR prove there cannot be one.

   (b) Either give an example of a Dhruv coloring $COL : \binom{\mathsf{N}}{2} \rightarrow \mathsf{N}$ that has an infinite min-homog set OR prove there cannot be one.

   (c) Either give an example of a Dhruv coloring $COL : \binom{\mathsf{N}}{2} \rightarrow \mathsf{N}$ that has an infinite max-homog set OR prove there cannot be one.

   (d) Either give an example of a Dhruv coloring $COL : \binom{\mathsf{N}}{2} \rightarrow \mathsf{N}$ that has an infinite rainbow set OR prove there cannot be one.

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3. (40 points) The following is a known theorem that we will refer to in two (very different) questions:

Theorem USETHIS For all k there exists G such that for all COL : $[G] \times [G] \rightarrow [2]$ there exists a, b, d such that all the elements of

$\{a, a+d, \ldots, a+(k-1)d\} \times \{b, b+d, \ldots, b+(k-1)d\}$ is monochromatic

($k = 2$ is the square theorem. $k = 3$ would be a regular 3×3 grid being mono.)

   (a) (20 point) Find a lower bound on G from **Theorem USETHIS** using the Prob Method.

   (b) (20 points) (You may and should use the **Theorem USETHIS**.) Show that there exists M such that, for all 2-colorings of

$$[M] \times [M] \times [M].$$

   there exists four points that are the same color of the following form:

   (x, y, z)
   $(x + d, y, z)$
   $(x, y + d, z)$
   $(x, y, z + d)$
   (Feel quite free to use pictures in your solution.)
   where $x, y, z \in [M]$ and $d \geq 1$.

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4. (20 points) Recall that

   $LR_2(k)$ is the least n such that any 2-coloring of $\binom{\{k,\ldots,n\}}{2}$ has a large homog set H where large means that $|H| > \min(H)$ (note the strict inequality). In class we showed $LR_2(2) \le 13$ and there are now slides on that on the course websites.

   Prove that $LR_2(2) \le 12$.

5. (20 points) For this problem you MAY NOT assume Rado's theorem. You will essentially be proving it in a particular case.

   Fill in an $A \ge 1$ and a $B \ge 1$ such that the following 2-part theorem is true, and prove both parts.

   (a) There exists a c and a c-coloring of N such that the equation

   $$w + 2x + 5y - Az = 0$$

   has NO mono solution. Try to make A as small as possible.

   (b) For every c there exists n such that for all c-colorings of $[n]$ there is a mono solution to the equation

   $$w + 2x + 5y - Bz = 0$$

   with w, x, y, z ALL DIFFERENT. Try to make B as large as possible. (YES I know that for this part I said w, x, y, z ALL DIFFERENT and for Part a I did not. This is NOT a mistake.)