**Homework 1, Morally Due Tue Feb 11, 2020**

COURSE WEBSITE:
`http://www.cs.umd.edu/~gasarch/COURSES/858/S20/index.html`
(The symbol before gasarch is a tilde.)

1. (0 points but if you do miss the midterm and don't tell Prof Gasarch about it ahead of time, it is $-100$ points) What is your name? Write it clearly. Staple your HW. When is the midterm tentatively scheduled (give Date and Time)? If you cannot make it in that day/time see me ASAP.

2. (20 points)

   (a) (10 points) Prove that for every $c$, for every $c$ coloring of $\binom{\mathsf{N}}{2}$, there is a homogenous set USING a proof similar to what I did in class.

   (b) (10 points) Prove that for every $c$, for every $c$ coloring of $\binom{\mathsf{N}}{2}$, there is an infinite homogenous set USING induction on $c$.

   (c) (0 points) Which proof do you like better? Which one do you think gives better bound when you finitize it?

3. (30 points) Prove the following theorem rigorously (this is the infinite $c$-color $a$-ary Ramsey Theorem):

   **Theorem 1.** *For all $a \geq 1$, for all $c \geq 1$, and for all $c$-colorings of $\binom{\mathbb{N}}{a}$, there exists an infinite set $A \subseteq \mathbb{N}$ such that $\binom{A}{a}$ is monochromatic ($A$ is an infinite homogeneous set).*

   The proof should be by induction on $a$ with the base case being $a = 1$.

4. (25 points) State and prove a theorem with the XXX filled in.

   For every coloring (any number of colors) of $XXX(n)$ points there is EITHER: (a) a set of $n$ that are all colored the same, or (b) a set of $n$ points that are all colored differently. However!- there IS a coloring of $XXX(n) - 1$ points such that there is NEITHER: (a) a set of $n$ that are all colored the same, or (b) a set of $n$ points that are all colored differently.

**THERE IS A PROBLEM ON THE NEXT PAGE**

5. (25 points)

Suppose $x_1, x_2, x_3, \ldots$ be an infinite increasing sequence of natural numbers. Let $p(y_1, y_2, \ldots, y_k)$ be any function on natural numbers, and let $q(z)$ be an increasing and unbounded function on the naturals.

Prove that there exists an infinite subsequence $y_1, y_2, \ldots$ such that for all $y_{i_1} < y_{i_2} < \cdots < y_{i_k} < y_{i_{k+1}}$, $p(y_{i_1}, \ldots, y_{i_k}) < q(y_{i_{k+1}})$.