Note: In this homework, $R(a,b)$ refers to the 2-ary asymmetric Ramsey numbers: $R(a,b)$ is the least $n$ such that every 2-coloring of $K_n$ has a mono RED $K_a$ or a mono BLUE $K_b$. Similarly, $R(a,b,c)$ is the 2-ary asymmetric Ramsey with three colors: $R(a,b,c)$ is the least $n$ such that every 3-coloring of $K_n$ has a monoc RED $K_a$ or a mono BLUE $K_b$ or a mono GREEN $K_c$.

1. (0 points) What is your name? Write it clearly.

2. (30 points) We never defined $R(1, b)$ or $R(a, 1)$. Define it so that the inequality

$$R(a, b) \leq R(a - 1, b) + R(a, b - 1)$$

holds. Does the definition also make intuitive sense?

3. (30 points) Let $R(a,b,c)$ be the least $n$ such that, for all 3-colorings of the edges of $K_n$, there exists either a RED $K_a$, a BLUE $K_b$, or a GREEN $K_c$.

   (a) Give an upper bound on $R(2, b, c)$ in terms of $R$ on two variables.
   
   (b) Recall In class we proved

   $$R(2, b) = b \text{ and } R(a, 2) = a$$

   $$R(a, b) \leq R(a - 1, b) + R(a, b - 1).$$

   We used these two relations to get upper bounds on $R(a, b)$. We did this by viewing the size of the input $(a, b)$ as $a + b$. With that viewpoint we are able to get upper bounds on $R(a, b)$ by using upper bounds on smaller pairs.

   In this problem you will use this approach for $R(a, b, c)$.

   The size of $(a, b, c)$ is $a + b + c$. State and prove an inequality that upper bounds $R(a,b,c)$ in terms of $R$ on smaller triples. (for example, it could be $R(a, b, c) \leq R(\lceil \sqrt{a} \rceil, b - 1, c) + R(\lfloor a/2 \rfloor, 2, 2)$ — but its not).
(c) Use parts (a) and (b) to determine reasonable upper bounds on 
\(R(2, 2, 2), R(2, 2, 3), R(2, 3, 3), \) and \(R(3, 3, 3)\).

4. (40 points) The following is a corollary of VDW’s theorem that we will 
cover later in the class

**VDW Theorem** For all \(k\) there exists \(W = W(k)\) such that for all 
2-coloring of \([W]\) there exists \(a, d\) such that

\[ a, a + d, \ldots, a + (k - 1)d \]

are all the same color.

(This is called a *monochromatic arithmetic progression of length* \(k\)
which we abbreviate *mono* \(k\)-AP.

Use the Prob Method to get a LOWER BOUND on \(W(k)\).