1. (0 points) What is your name? Write it clearly.

2. (30 points) **Definition:** Let $X$ be any set of naturals (it can be finite or infinite) A coloring $COL : \binom{X}{2} \to \omega$ is *Erika* if $COL(x, y) \leq \min\{x, y\}$. (Note that $\omega = \{1, 2, 3, \ldots\}$, so $COL(1, y) = 1$ always.)

Consider the following (true) statement which we call STATEMENT.

*If COL is an Erika coloring of $\binom{N}{2}$ then either (1) there exists an infinite homog set, or (2) there exists an infinite min-homog set.*

(a) (10 points) Prove STATEMENT from the Can Ramsey Theory.

(b) (10 points) Prove STATEMENT directly, NOT using the Can Ramsey Theory.

(c) (10 points) Formulate a finite version of STATEMENT. Give a proof of your statement. It DOES NOT have to give bounds on $n$.

3. (25 points) Prove the following:

*For all $k$ there exist $n$ such that for all Erika colorings $COL : \binom{\{k, \ldots, n\}}{2} \to \omega$ there exist either (1) a LARGE homog set, or (2) a LARGE min-Homog set. (You need not get a bound on $n$.)*

4. (25 points)

(a) (15 points) Prove the following. There exists a function $f$ such that the following holds:

*If $T_1, T_2, \ldots, T_{f(k)}$ is a FINITE sequence of trees, where $T_i$ has at most $2^{k_i}$ nodes, there is an uptick.*

For this problem the trees are ordered as $T_1 \leq T_2$ if $T_1$ is a minor of $T_2$.

(b) (10 points) Is there some function $g(i, k)$ such that if you replace the $2^{k_i}$ in the first question with $g(i, k)$. the theorem is now false?
5. (20 points) Let $c_1, c_2, c_3 \in \mathbb{N}$.
   Let $\text{COL}_1$ be a $c_1$-coloring of $\binom{\mathbb{N}}{1}$.
   Let $\text{COL}_2$ be a $c_2$-coloring of $\binom{\mathbb{N}}{2}$.
   Let $\text{COL}_3$ be a $c_3$-coloring of $\binom{\mathbb{N}}{3}$.
   A set $H \subseteq \mathbb{N}$ is Nathan homogeneous if
   $\text{COL}_1$ restricted to $\binom{H}{1}$ is monochromatic, and
   $\text{COL}_2$ restricted to $\binom{H}{2}$ is monochromatic, and
   $\text{COL}_3$ restricted to $\binom{H}{3}$ is monochromatic.

   (a) (10 points) Show that for all $c_1, c_2, c_3$, for all $c_1$-colorings of $\binom{\mathbb{N}}{1}$,
       $c_2$-colorings of $\binom{\mathbb{N}}{2}$, and all $c_3$-colorings of $\binom{\mathbb{N}}{3}$, there is an infinite
       Nathan Homogenous set.

   (b) (10 points) State and prove a finite version of part a, with bounds.
       (You may use the known bounds on Ramsey Numbers.)