Two Triangles

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Lets Party Like Its January of 2019

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Pedagogical Point In this class we then did hypergraphs so we stated the above theorem in terms of coloring $\binom{[6]}{2}$. In this talk it will be helpful to use graphs.

Trivial Theorem

For all 2-cols of edges of K_{12} there are 2 mono K_3's

Question Find n such that

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1. For all 2-coloring of the edges of K_6 there are 2 mono K_3's
2. There exists a 2-coloring of the edges of K_5 that does not have 2 mono K_3's.

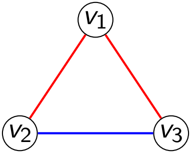
Proof of K_6 Two Triangles Theorem

Theorem For all 2-cols of edges of K_6 there are 2 mono K_3's
Proof Let COL be a 2-coloring of the edges of K_6.
Let R, B, M, be the SET of RED, BLUE, and MIXED triangles.

$$|R| + |B| + |M| = \binom{6}{3} = 20.$$

We show that $|M| \leq 18$, so $|R| + |B| \geq 2$.

A Mixed Triangle Has a Vertex Such That



- (v_2, v_1) is red, (v_2, v_3) is blue. View this as $(v_2, \{v_1, v_3\})$.
- (v_3, v_1) is red, (v_3, v_2) is blue. View this as $(v_3, \{v_1, v_2\})$.

We could map every mixed triangle to $\binom{V \times \binom{V}{2}}{2}$ but that would be a mess and not quite right anyway.

Map ZAN to M

Definition A **Zan** is an element $(v, \{u, w\}) \in V \times \binom{V}{2}$ such that $v \notin \{u, w\}$ and $COL(v, u) \neq COL(v, w)$. ZAN is the set of Zan's.

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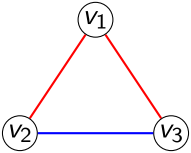

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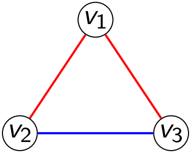

$(v_2, \{v_1, v_3\})$ and $(v_3, \{v_1, v_2\})$.

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So there are at least 2 Mono Triangles.