1 Open Problems

Here are some open problems in this area:

- Are DIAM and APSP subcubic equivalent?
- Is it possible to get a lower bound for approximating RADIUS and MEDIAN similar to the one for DIAM?

2 Put at beginning when discussing APSP

1. Chan & Williams [?] found an algorithm for APSP in deterministic time $\frac{n^3}{2^{\Omega(\sqrt{\log n})}}$ time, matching the previous known randomized algorithm.

2. Bringmann et al. [?] found an efficient algorithm for approximating APSP. Formally the problem is as follows. Fix $\epsilon > 0$. Given an undirected unweighted graph G, return for each pair of vertices, a path which is at most $(1+\epsilon) \times \text{OPT}$. They have an $O(\frac{n^\omega}{\epsilon}\text{polylog}(\frac{n}{\epsilon}))$ algorithm where ω is the exponent for matrix multiplication (currently $\omega \sim 2.37$).

3 Further Results

1. The Tree Edit Problem is (informally) as follows: Given two trees, what is the least number of changes needed to get one from the other. Bringmann et al. [?] show the this problem is APSP-hard.

2. The Matrix Product Verification Problem is as follows: Given matrices A, B, C verify that $AB = C$ where the product is over the $(\min, +)$-semiring. Williams & Williams [?] showed this problem is subcubic equivalent to APSP.

3. The Replacement Paths Problem is as follows: Given weighted directed graph G, vertices s, t, and a shortest (s,t)-path P compute the length of the shortest (s,t)-path that does not use any edge from P. Williams & Williams [?] showed this problem is subcubic equivalent to APSP.

4. The Metricity problem is as follows: Given an $n \times n$ nonnegative matrix A, determine whether it defines a metric space on $[n]$, i.e. if A is symmetric, has 0s on diagonal and entries satisfy the triangle inequality. Williams & Williams [**?**] showed this problem is subcubic hard.

5. The All-Pairs Min Cut Problem is as follows Given a graph G compute, for every pair of vertices s, t, a min $(s - t)$ cut. Abboud et al. [**?**] showed that this problem has a super-cubic lower bound of $n^{\omega-1-o(1)}k^2$ from a reduction from 4-clique (a novel reduction instead of APSP)

6. The Dynamic shortest paths problem preprocess a planar graph G such that insertions/deletions of edges are supported as well as distance queries between two nodes u, v assuming the graph is planar at all time steps. Abboud & Dahlgaard [**?**] showed that, assuming the APSP-hypothesis is true then, for all $\epsilon > 0$, this problem cannot be solved in time $O(n^{\frac{1}{2}-\epsilon})$.

The next two problems have as their hypothesis that the unweighted APSP problem is hard. Both reasults are by Linoln et al. [**?**]

1. The All Edges Monchromatic Triangle Problem is as follows. Given an n-node graph G with edges labelled a color from 1 to n^2, decide for each edge if it belongs to a monocrhomatic triangle, a triangle whose 3 edges have the same color. If this problem has a $T(n)$ time algorithm then the unweighted APSP has an $O(T(n)\log^n)$ time algorithm.

2. The Min-Max Product Problem is as follows. Given two matrices A, B compute the min max matrix C where $C_{i,j} = \min_k \max(A_{ik}, B_{kj})$. If this problem has a $T(n)$ algorithm then the Unweighted APSP problem has a $O(T(n)\log n)$ time algorithm.