BILL AND NATHAN, RECORD LECTURE!!!!

BILL RECORD LECTURE!!!

Lower Bounds on Approx Clique
Via PCP and Gaps

Notation for Size of Max Clique

If G is a graph then

$$\omega(G) = \text{the size of the max clique in } G.$$

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   No. We will not quite show this but will show something close.

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$A \in \mathrm{PCP}(c \lg n, d \lg n, \frac{1}{n})$.
We use the following in a poly time program for A:

1. The approx which gives $\geq n^{-\delta}\omega(G)$.
2. The $(c \lg n, d \lg n, \frac{1}{n})$ PCP for A.

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1) $(\exists)$ a query that they answer differently. **Inconsistent**
2) $(\forall)$ queries in common they answer the same. **Consistent**

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   In order to make these two cases not overlap we need

$$d - 1 < d - (c + d)\delta$$

$$\delta < \frac{1}{c + d}$$

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3. By our comments, no other case will occur.

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5) We now turn to a SAT-like non-approx result.