Dominion End Games
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Dominion is a fun deck-building game. One can play it either in person or online.

dominion.games

We will investigate the following types of questions. We give some examples; however, they might not make sense if you don’t know that game.

1. What are good opening buys? For example, if you buy a Moneylender and a Silver in the first two turns, what is the probability that you can buy a Gold in the third or fourth turn. Or more generally, if you buy cards \(x\) and \(y\) in the first two turns what is the prob you will be able to buy a card cost \(z\) in the third or fourth turn. These questions can all be done by simulation. Some can be done analytically as well, but even those will need a program.

2. Certain combinations of cards lead to strategies. Test how good they are. For example, if you have 7 Inventors and 5 Laboraties, how many turns before you can get all the Provinces. More generally, given a combination of cards and a strategy, how many turns (in Soliatire Dominion) before you can get \(x\) points. These questions can all be done by simulation. Solving them analytically might require Markov Chains.

3. End Games: There are times you need to know if your opponent can win on the next move. Example: If your opponent has 3 Golds, a Market, and 4 Provinces, if they get five random cards what is the probability they can buy a Province. More generally, if your opponent has BLAH cards, what is the prob they can buy a Province.

MORE DETAIL BELOW, DO NOT HAVE TO READ.
If you do not know the game Dominion then read no further.
Imagine the following scenario:

1. You are in a 2-player game so there are 8 Provinces total.

2. Your opponent is ahead by 2 Victory Points.

3. You have 10 cards. Your hand is 2 Golds, 1 Silver, and 2 Provinces. The 5 cards you in your deck are 3 Golds, 1 Silver, and 1 Province.
4. Your opponent has 10 cards: 3 provinces, 3 Gold, 2 estate, 2 coppers. You do not know what is in his hand.

5. It is your turn. You might think *OH, I’ll buy a province* Is this a good move?

   - If it is likely your opponent can also buy a province than its a bad move: you buy one, he buys one, and then the game is over and he wins. You might be better off buying a duchy and then buy a province the following turn. Then its you but Duchy, He buys Province, You buy Province, and you win. (This case is easy since you KNOW you can buy a Province the next turn.)

   - If it is likely your opponent cannot buy a province, then buy one.

This is an example of a Dominion End-Game.

You want to determine the probability that your opponent can buy a province.

In a more complicated scenario you may need to figure out the prob that YOU can buy a province in the following turn.

The project is to

1. Write a program that determines probabilities in Dominion End Games and hence can determine optimal play.

2. We will extend this to more complicated cards and actions, but still keep it in the end game.

3. Much harder- extend to play the game itself.