Ramsey Theory TO DO

by William Gasarch

Ariel and Xinhe (and I think others) have told me that they are more interested in Ramsey Theory than in ML. I said

Once we get to a good turning point in ML we can look more at Ramsey.

I was thinking

Once we have an ML playing VDW(4) very well, we can look more at Ramsey.

I am confident this will happen, but perhaps not so soon as to make it the turning point.

Below I list what I want you to READ in Ramsey Theory by next week Thursday, and also a RESEARCH project you can begin NOW in Ramsey Theory.

For below readings I am referring to the Ramsey Course Website:

`https://www.cs.umd.edu/users/gasarch/COURSES/858/S20/notes.html`

READING

1. Read NOTES: Primitive Recursive stuff, but just Section 1.
2. Read NOTES: The Large Ramsey Theorem. The proof I give for Large Ramsey does not give any bounds on the Large Ramsey Numbers. Try to find a proof that will give bounds.
3. Read SLIDES: LR(2) LESS THAN OR EQUAL 13. This is a proof that Large Ramsey of 2 exists and gives a bound.

RESEARCH Recall Rado's Theorem:

Theorem 0.1 *Let $b_1, \ldots, b_n \in \mathsf{Z}$. The following are equivalent.*

- *For all finite colorings of N there exists $x_1, \ldots, x_n$ the same color such that $\sum_{i=1}^{n} b_i x_i = 0$.*
- *Some subset of the b_i's sums to 0.*

Lets look at an example. Take

$$w + 2x + 5y - 9z = 0$$

No subset of $1, 2, 4, -8$ sums to 0. Hence there is a finite coloring of N with no mono solution. From the proof we can get a coloring: let p be the smallest prime that is not equal to any of the abs value of any of the sums.

The sums are: $1, 2, 3, 5, 6, 7, 8, 9$. The smallest prime bigger than all of these is 11. The proof of Rado gives us that there is a 10-coloring, namely

$$COL(11^a b) = b \pmod{11}$$

So is the question of

$$w + 2x + 5y - 9z = 0$$

and coloring done? NO- look at what we know:

There is a 10-coloring of N with no mono solution

What about a 9-coloring? an 8-coloring? ... a 2-coloring? I suspect that a 2-coloring will yield a mono solution, but I actually do not know.

Research Project Look at equations like the one above. Look at what happens when the number of colors is LESS THAN the number given by Rado. What happens? You can try to use math on it OR you can write a program Or you can do both. NOTE: I do not know the answers here.