I forgot to say that it is very interesting and the puzzles are addictive!

Seems like I found a way to "prove" FORCE-5 with a bit more work. But I may be wrong!

1. (A) Say $\text{COL}(x) = R$ and $\text{COL}(x + 9) = B$ (they must be different)

2. (A) IMPLIES $\text{COL}(x + 1)$ and $\text{COL}(x + 4)$ are in $\{B, G\}$

3. (A) IMPLIES $\text{COL}(x + 5)$ and $\text{COL}(x + 8)$ are in $\{R, G\}$

4. by FORCE-7, (A) IMPLIES $\text{COL}(x + 7) = R$ and $\text{COL}(x + 2) = B$

5. Now, $\text{COL}(x + 1)$ in $\{B, G\}$ and $\text{COL}(x + 2) = B$ implies

$$\text{COL}(x + 1) = G(\text{since they must be different}).$$

6. Now, $\text{COL}(x + 8)$ in $\{R, G\}$ and $\text{COL}(x + 7) = R$ implies

$$\text{COL}(x + 8) = G(\text{idem}).$$

7. $\text{COL}(x + 1) = G$ and $\text{COL}(x + 5)$ in $\{R, G\}$ implies

$$\text{COL}(x + 5) = R = \text{COL}(x)(\text{since they must be different}).$$

8. $\text{COL}(x + 8) = G$ and $\text{COL}(x + 4)$ in $\{B, G\}$ implies

$$\text{COL}(x + 4) = B = \text{COL}(x + 9)(\text{idem}).$$

So we have FORCE-5 as a consequence of FORCE-7. It is interesting from a logical perspective! That kind of reasoning used a decomposition $p(x) = p(y) + p(z) + p(t)$, here a square as a sum of three squares. That may be interesting to see what we get from that kind of decomposition.