1 PROJECT ONE: Random Ramsey Theory

I thought I assigned this a while back as a TO DO but perhaps I did not. So now I will make it a PROJECT.

1. General Pitch 1: We know that $R(3) = 6$. But if you color the edges of K_5 at random then the prob that you DO NOT get a mono K_3 is very small. What about other Ramsey Numbers?

2. General Pitch 2: We know that $R(3) = 6$. But if you color the edges of K_6 at random then HOW MANY mono K_3's will you get? What about other Ramsey Numbers?

3. Write a program that does the following: Given n, k

   1) Randomly color the edges of K_n and count the number of mono K_k's there are.

   2) DO THAT 100 times (maybe less if this is to much for your computer).

4. Recall that $R(3) = 6$. That motivates the following experiment:

   a) If you 2-color the edges of K_5 100 times Collect the data and see which number-of-K_3's occurs the most times.

   b) If you 2-color the edges of K_6 100 times Collect the data and see which number-of-K_3's occurs the most times (it will be at least 1).

   c) If you 2-color the edges of K_7 100 times Collect the data and see which number-of-K_3's occurs the most times (it will be at least 1).

   $\vdots$

   Stop at K_{10}.

   Make conjectures about the following:

   i) If you randomly color the edges of K_n then the expected number of mono K_3's is XXX (for large n).

   ii) If you color the edges of K_n then you will always get at least XXX mono K_3's. (for large n).

5. Recall that $R(4) = 17$. That motivates the following experiment:

   a) If you 2-color the edges of K_{10} 100 times Collect the data and see which number-of-K_4's occurs the most times.

   b) If you 2-color the edges of K_{11} 100 times Collect the data and see which number-of-K_4's occurs the most times (it will be at least 1).

   c) If you 2-color the edges of K_{12} 100 times Collect the data and see which number-of-K_3's occurs the most times (it will be at least 1).

   $\vdots$

   Stop at K_{25}.

   Make conjectures about the following:

   i) Find the m such that the following two hold:

If you color the edges of K_m at random then the prob of getting a mono K_4 is ≥ 0.9.

If you color the edges of K_{m-1} at random then the prob of getting a mono K_4 is < 0.9.

ii) If you randomly color the edges of K_n then the expected number of mono K_4's is XXX (for large n).

iii) If you color the edges of K_n then you will always get at least XXX mono K_4's (for large n).

6. I will teach you other Ramsey Theorems that will lead to similar questions.

2 Project TWO: Ramsey Numbers via SAT Solvers

1. Investigate publicly available SAT solvers and what format they use. Try them out.
2. Phrase questions like

   There is a 2-coloring of the edges of K_{13} with no mono K_4 as a SAT question. Try the SAT solvers on them.

3. Alternatively: Write your own SAT solvers and see how they do.
4. I have some other ideas about this topic as well.