Conditional Probability and Bayes Theorem

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Want to formalize this intuition so we can apply it to more complicated scenarios.

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2 elements in the desired outcome: $\{1, 3\}$.
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So the prob is $\frac{2}{3}$.

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How many: $1+2+3+4+5+6=21$.

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Think About Could a diff assumption make the prob go up?

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(Formal) $\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$.

This is exactly what I have been doing in the examples.

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$\Pr(B \mid A) = \frac{\Pr(A|B)\Pr(B)}{\Pr(A)}$.

Bayes's theorem

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$\Pr(A \mid B) = \Pr(B \mid A) \cdot \frac{\Pr(A)}{\Pr(B)}$

Note: This is very useful in both this course and in life.

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Is the coin definitely biased?

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Is the coin definitely biased? No.

What is Prob that it is biased? VOTE:

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2. Between 0.98 and 0.99
3. Between 0.97 and 0.98
4. Less than 0.97

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We will see that it is 0.982954, so between 0.98 and 0.99.

Example of Application of Bayes's theorem

$$\mathrm{Pr}(B|H^{10}) = \frac{\mathrm{Pr}(B)\mathrm{Pr}(H^{10}|B)}{P(H^{10})}$$

$\mathrm{Pr}(B) = \frac{1}{2}$

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$= \Pr(H^{10}|F)\Pr(F) + \Pr(H^{10}|B)\Pr(B) = \frac{1}{2}\left((\frac{1}{2})^{10} + (\frac{3}{4})^{10}\right)$

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Put it together to get

$$\Pr(B|H^{10}) = \frac{1}{1 + (2/3)^{10}} = 0.982954.$$

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$$\mathrm{Pr}(B|H^{10}) = \frac{\mathrm{Pr}(B)\mathrm{Pr}(H^{10}|B)}{P(H^{10})}$$

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$$\mathrm{Pr}(B|H^{n}) = \frac{1}{1 + (2/3)^{n}}.$$