Poker Problems

1. Show that

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

with a combinatorial proof.

(Hint: Show that the RHS solves the question of how many ways to choose k objects out of n objects.)

2. By convention $(\forall n \ge 0)[\binom{n}{0} = 1]$ and $(\forall k \ge 1)[\binom{0}{k} = 0]$.

From Part 1 that $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.

Use these two equations to write a program that will do the following:

Given N, K outputs $\binom{k}{n}$ for all $0 \le k \le K$ and $0 \le n \le N$.

Run this program for $N = 52$ and $K = 6$.

3. For this problem we use the normal deck: 13 ranks 4 suites. What is the probability of getting a pair but not better- so NOT two pairs.

4. Let $r, s \in \mathbb{N}$. Assume you have a deck of cards with ranks in $\{1, \ldots, r\}$ and suites in $\{1, \ldots, s\}$. Let d be the number of cards dealt. (For a normal deck $r = 13$, $s = 4$, $d = 5$.)

A Straight Flush is d cards of the same suite that are in order. We DO allow wrap-around.

A Straight is d cards in order that is NOT a straight flush.

A Flush is d cards of the same suite that is NOT a straight flush

   (a) What is the probability of a straight flush as a function of r, s, d.
   (b) What is the probability of a straight as a function of r, s, d.
   (c) What is the probability of a flush as a function of r, s, d.
   (d) Are there an values of (r, s, d) where the probability of flush is higher than that of a straight?

5. Oliver-Poker is poker with a normal deck, but each player gets SIX cards.

   For all questions here answer it BOTH in terms of binomial coefficient (e.g.,

$$\frac{\binom{12}{8}}{\binom{52}{6}}$$

)

   and as an actual number to four places (e.g., 0.2192).

   (For that you will use the output of the program you wrote in Problem 2.)

   (a) (10 points) What is the probability of getting three 2-of-a-kinds?

      Example: $(2\heartsuit, 2\spadesuit, 4\heartsuit, 4\diamondsuit, 8\spadesuit, 8\clubsuit)$ is three 2-of-a-kind.

      Counterexample: $(2\heartsuit, 2\spadesuit, 2\clubsuit, 2\diamondsuit, 8\spadesuit, 8\clubsuit)$ DOES NOT count as three 2-of-a-kind.

   (b) (10 points) What is the probability of getting a 2-of-a-kind and a 4-of-a-kind?

      Example: $(2\heartsuit, 2, 2\heartsuit, 2\diamondsuit, 8, 8\clubsuit)$ is a 4-of-a-kind and a 2-of-a-kind.

   (c) (10 points) What is the probability of getting a two 3-of-a-kind?

      Example: $(2\heartsuit, 2\spadesuit, 2\heartsuit, 8\diamondsuit, 8\spadesuit, 8\clubsuit)$ is two 3-of-a-kind.