Comments on
Gasarch Primes 8-7-26

I will present a big structural correction that involves re-arranging things, and then also some corrections on what you actually wrote. DO the corrections on what you actually wrote FIRST since once you change things around it may be hard to find out which part I was pointing to.

1. Page 1. Title. You need a better title. This goes along with the next comment.

2. Overall Comment. The theme should Elementary proofs that subsets of the primes are infinite. that could also be the title
Elementary Proofs that Subsets of the Primes and Infinite

   With that in mind, the paper should be structured as follows

   (a) **Introduction**

   Your first paragraph should be something like the following:
   By Dirichlet's theorem:
   For all a, b, $gcd(a, b) = 1$, the set

$$\{p\colon p \text{ is prime and } p \equiv a \pmod{b}\}$$

   is infinite.
   This proof is difficult. In this paper we will provide elementary proofs of subcases.

   (b) **Definitions and Examples**

   Define Integral Domain, units, primes, irreducibles, UFD, Euclidean domains.
   Give examples and counterexamples.
   Examples of primes. Problem 7, 8 and also do $\mathsf{Z}[i]$, $\mathsf{Z}[\sqrt{2}]$.
   Examples units- Problem 7, 8, use $\mathsf{Z}[i]$ and $\mathsf{Z}[\sqrt{2}]$
   Example of UFD and NON-UFD (Problems 2 and 3)
   Examples of when primes and irred are the same and when they are not.

   (c) **The Set of Primes $\equiv a \pmod 4$ where $a \in \{1, 4\}$ is Infinite**

(d) **The Set of Primes** $\equiv a \pmod 5$ **where** $a \in \{1,2,3,4\}$ **is Infinite**

(e) **The Set of Primes** $\equiv a \pmod 6$ **where** $a \in \{1,5\}$ **is Infinite**

(f) **Limitations to our Methods**
Discuss the difficulties you have getting a result for mod 7.

3. Page 1. Lemma 0.1. The word *multiples* means multiples by an integer. You should clarify that its multiples by an elements of $\mathsf{Z}[\sqrt{-k}]$.

4. Page 2. Lemma 0.4. You should say that $\mathsf{Z}[\sqrt{-k}]$ is a Euclidean Domain USING the norm N.

5. Page 2. In your note after the proof of Lemma 0.4 you should clarify that for $k \geq 3$ the norm would not work. Also see next point.

6. Page 2. Your note after the proof of Lemma 0.4 only shows that THIS norm would not work. How about another norm? Do you have a proof that if $k \geq 3$ then $\mathsf{Z}[\sqrt{-k}]$ is NOT a Euclidean domain–if thats true. You later show that $\mathsf{Z}[\sqrt{-5}]$ is not a UFD and hence is not an ED. But what about (say) $\mathsf{Z}[\sqrt{-7}]$?

7. Page 3. You use the theorem that if $p \equiv 1 \pmod 4$ then -1 is a square. Thats fine. But you should make that into a Lemma (which you don't have to prove) and then refer to the Lemma in your proof.

8. Page 3. Problem 5. there is no space between mod and 5.

9. Page 3. Problem 5. The title indicates that you will show that

   The Set of Primes $\equiv a \pmod 5$ **where** $a \in \{1,2,3,4\}$ **is Infinite**

   but the section seems to only prove this for $a = 1, 4$.

10. I have not read the proofs carefully. I will do that for the next version you send me.