TO DO LIST

Exposition by William Gasarch

June 8, 2026

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Characterize and proof exactly which Gauss ints are primes.

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Primes infinite

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Prove that the set of primes $\equiv 4 \pmod{5}$ is infinite.

$\{a + b\sqrt{2} \colon a, b \in \mathbb{Z}\}$

Let $\mathbb{D} = \{a + b\sqrt{2} \colon a, b \in \mathbb{Z}\}$.
If $x = a + b\sqrt{2}$ then let $\overline{x} = a - b\sqrt{2}$.
Let $N(x) = x\overline{x}$.
The proof I gave that $\overline{xy} = \overline{x}\ \overline{y}$ did not need that I used $-d$.
So we still have $N(xy) = N(x)N(y)$.
Easy to proof that $N(x) \in \{-1, 1\}$ iff $x \in \mathbb{U}$.
FIND ALL THE UNITS IN $\mathbb{D}$.

The Algebraic Integers

Let A be the set of all reals that are roots of a polynomial in $\mathbb{Z}[x]$ that has lead coefficient 1.
For example, $\sqrt{2} \in A$ since it satisfies $x^2 - 2 = 0$.
But $\frac{1}{2} \notin A$ since the only poly it satisfies is $2x - 1 = 0$.
Find all units and primes in A.

Not Quite the Rationals

Let p be a prime.
Let $B = \{\frac{a}{b} \colon b \not\equiv 0 \pmod{p}\}$.
Find all units and primes in B.