TO DO on XK project

1 Weakining the Hypothesis

O In the *Weakinging the Hypothesis* project you are actually NOT Weakening the hypothesis, you are showing that it can't be done by giving countrexamples

1. You have data for a, b, c, d, e, f all between -4 and 4. See if you can extend to 5 then 6, stop when its infeasible.

   The number of counterexamples will go up as you can use more values, but does it go up much?

2. Try to spot a pattern so that you can have a theorem that gives and infinite number of counterexamples. PROVE that theorem!

3. Try to characterize all of the quadratic counterexamples.

4. You have been doing

   $x + y \in \mathbb{Z}$ but $x^2 + y^2 \notin \mathbb{Z}$

   and

   $x + y \in \mathbb{Z}$ and $x^2 + y^2 \in \mathbb{Z}$ but $x^3 + y^3 \notin \mathbb{Z}$.

   Work on

   $x + y \in \mathbb{Z}$ and $x^2 + y^2 \in \mathbb{Z}$ and $x^3 + y^3 \in \mathbb{Z}$ but $x^4 + y^4 \notin \mathbb{Z}$.

   and also ALL combinations of these four things being in or out of $\mathbb{Z}$.

2 Strengtheining the Hypothesis

In the *strengthening the Conclusion* project you are actually NOT strenthening the conclusion, you are showing that it can't be done by giving countrexamples

1. Characterize the counterexamples.

   You have shown that $x = \sqrt{c}$, $y = -\sqrt{c}$ is a counter example

   Find all counterexamples of the form $a + b\sqrt{c}$ where $a, b \in \mathbb{Z}$ and $c \in \mathbb{N}$ (and not a square).

2. For all natural numbers $r \geq 2$ see what counterexamples there are where x, y are both of the form $a + bc^{1/r}$.

3. Come up with conjectures, and then try to prove them!

3 Different Domains

You are looking at statements of the form

$(\forall x, y \in \mathbb{D}_1)[(x + y \in \mathbb{D}_2, \ldots, x^k + y^k \in \mathbb{D}_2) \implies (\forall i \in \mathbb{N})[x^i + y^i \in \mathbb{D}_3]$.

You are looking at $\mathbb{D}_1, \mathbb{D}_2, \mathbb{D}_3$ being any of $\mathbb{C}, \mathbb{R}, \mathbb{Q}, \mathbb{Z}$

Make a chart of the nontrivial $\mathbb{D}_1, \mathbb{D}_2, \mathbb{D}_3$. For each such triple give the lowest k that works OR say there is no such k. If you claim k is lowest, show that $k - 1$ does not work.