Questions that Arise from the $x^n + y^n$ Theorem

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Example that uses Notation!
$(\forall n \in \mathbb{N})(\exists x, y \in \mathbb{R})$
$[(x + y \in \mathbb{Z}) \wedge (x^n + y^n \in \mathbb{Z}) \wedge (x^{n+1} + y^{n+1} \notin \mathbb{Z})]$.

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For more terminology of this type see my blog post:
`https://blog.computationalcomplexity.org/2023/08/theorems-and-lemmas-and-proofs-oh-my.html`

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They are on the next slides.

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I suspect all are false. However, try to find the set of all counterexamples.

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If TRUE then find OPTIMAL k

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Of the 64 many will be trivial or follow from the main theorem.

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There are 5 people. I want to have 2 on each project so Alan will be on 2 of them.