TEST [2]

Skolem, Mahler, and Lech independently showed that if a sequence of numbers is generated by a linear difference equation then, with a finite number of exceptions, the places where the sequence is 0 is periodic. Hence the problem we are considering is called SKOLEM'S PROBLEM even though Skolem did not ask it.

Halava et al. [3] and Ouakine & Worrell [4] have nice surveys of what is known. (See those surveys for references). For $k = 1$ and $k = 2$ Skolem's problem is decidable. This is folklore and fairly easy. For $k = 3$ and $k = 4$ Skolem's problem is decidable. These proofs are difficult. For all $k \geq 5$ the problem is open. Halvana et al. claimed to show that for $k = 5$ the problem is decidable; however, Ouakine & Worrell showed that there was a bug in the proof.

Bilu et al. [1] showed that, assuming two conjectures in number theory, (1) the special case of the problem where the polynomial XXX has all roots of multiplicty 1, the problem is decidable , and (2) the $k = 5$ case is decidable.

# References


[1] Y. Bilu, F. Luca, J. Nieuwveld, J. Ouaknine, D. Purser, and J. Worrell. Skolem meets schanuel. In S. Szeider, R. Ganian, and A. Silva, editors, *47th International Symposium on Mathematical Foundations of Computer Science, MFCS 2022, August 22-26, 2022, Vienna, Austria*, volume 241 of *LIPIcs*, pages 20:1–20:15. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2022. https://doi.org/10.4230/LIPIcs.MFCS.2022.20. 1

[2] V. Blondel and N. Portier. The presense of a zero in an integer linear recurrent sequence is NP-hard to decide. *Linear Algebra and its applications*, 351–352:91–98, 2002. 1

[3] V. Halava, T. Harju, mika Hirvensalo, and J. Karhumaki. Skolem's problem—on the border between decidable and undecidable. Technical Report 683, Turku Centre for Computer Science, 2005. http://oldtucs.abo.fi/publications/view/?pub_id=tHaHaHiKa05a. 1

[4] J. Ouaknine and J. Worrell. Decision problems for linear recurrence sequences. In A. Finkel, J. Leroux, and I. Potapov, editors, *Reachability Problems - 6th International Workshop, RP 2012, Bordeaux, France, September 17-19, 2012. Proceedings*, volume 7550 of *Lecture Notes in Computer Science*, pages 21–28. Springer, 2012. https://doi.org/10.1007/978-3-642-33512-9_3 and https://people.mpi-sws.org/~joel/publications/positivity12.pdf. 1