The Muffin Problem

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How it Beganz

A Recreational Math Conference
(Gathering for Gardner)
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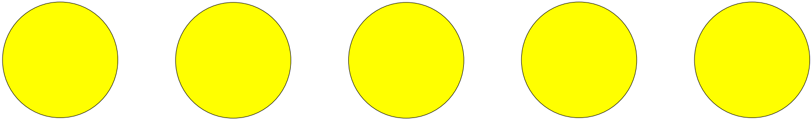
I found a pamphlet:

The Julia Robinson Mathematics Festival:
A Sample of Mathematical Puzzles
Compiled by Nancy Blachman

which had this problem, proposed by Alan Frank:

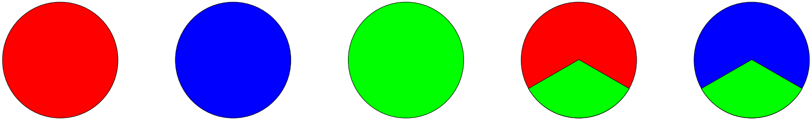
How can you divide and distribute 5 muffins to 3 students so that every student gets $\frac{5}{3}$ *where nobody gets a tiny sliver?*



5 Muffins, 3 Students, Proc by Picture

Person	Color	What they Get
Alice	**RED**	$1 + \frac{2}{3} = \frac{5}{3}$
Bob	**BLUE**	$1 + \frac{2}{3} = \frac{5}{3}$
Carol	GREEN	$1 + \frac{1}{3} + \frac{1}{3} = \frac{5}{3}$

Smallest Piece: $\frac{1}{3}$



Can We Do Better?

The smallest piece in the above solution is $\frac{1}{3}$.

Is there a procedure with a larger smallest piece?

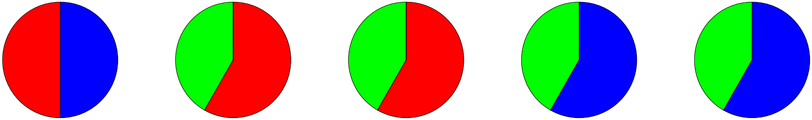
Work on it with your neighbor

5 Muffins, 3 People–Proc by Picture

YES WE CAN!

Person	Color	What they Get
Alice	**RED**	$\frac{6}{12}+\frac{7}{12}+\frac{7}{12}$
Bob	**BLUE**	$\frac{6}{12}+\frac{7}{12}+\frac{7}{12}$
Carol	GREEN	$\frac{5}{12}+\frac{5}{12}+\frac{5}{12}+\frac{5}{12}$

Smallest Piece: $\frac{5}{12}$



Can We Do Better?

The smallest piece in the above solution is $\frac{5}{12}$.

Is there a procedure with a larger smallest piece?

Work on it with your neighbor

5 Muffins, 3 People–Can't Do Better Than $\frac{5}{12}$

NO WE CAN'T!

There is a procedure for 5 muffins,3 students where each student gets $\frac{5}{3}$ muffins, smallest piece N. We want $N \le \frac{5}{12}$.

Case 0: Some muffin is uncut. Cut it $(\frac{1}{2}, \frac{1}{2})$ and give both $\frac{1}{2}$-sized pieces to whoever got the uncut muffin. (Note $\frac{1}{2} > \frac{5}{12}$.) Reduces to other cases. (**Henceforth:** All muffins cut into $\ge \mathbf{2}$ pieces.)

5 Muffins, 3 People–Can't Do Better Than $\frac{5}{12}$

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Case 1: Some muffin is cut into ≥ 3 pieces. Then $N \leq \frac{1}{3} < \frac{5}{12}$. (**Henceforth:** All muffins cut into 2 pieces.)

5 Muffins, 3 People–Can't Do Better Than $\frac{5}{12}$

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Case 1: Some muffin is cut into ≥ 3 pieces. Then $N \le \frac{1}{3} < \frac{5}{12}$. (**Henceforth:** All muffins cut into 2 pieces.)

Case 2: All muffins are cut into 2 pieces. 10 pieces, 3 students: **Someone** gets ≥ 4 pieces. He has some piece

$$\le \frac{5}{3} \times \frac{1}{4} = \frac{5}{12} \qquad \text{Great to see } \frac{5}{12}$$

What Else Was in the Pamphlet?

The pamphlet also had asked about

1. 4 muffins, 7 students.
2. 12 muffins, 11 students.
3. a few others

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There can't be much more to this.

If there is not much more to this then how come

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- Find a technique that solves many problems (e.g., Floor-Ceiling).

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- Find a technique that solves many problems (e.g., Floor-Ceiling).
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- Come across a problem where the techniques do not work.
- Find a new technique **which was interesting**.
- Lather, Rinse, Repeat.

General Problem

$\boldsymbol{f(m,s)}$ be the smallest piece in the best procedure (best in that the smallest piece is maximized) to divide m muffins among s students so that everyone gets $\frac{m}{s}$.

We have shown $f(5,3) = \frac{5}{12}$ here.

We have shown $f(m,s)$ exists, is rational, and is computable using a Mixed Int Program (in paper).

Amazing Results!/Amazing Theorems!

1. $f(43,33) = \frac{91}{264}$.
2. $f(52,11) = \frac{83}{176}$.
3. $f(35,13) = \frac{64}{143}$.

All done by hand, no use of a computer
by Co-author Erik Metz is a muffin savant !

Have **General Theorems** from which **upper bounds** follow.
Have **General Procedures** from which **lower bounds** follow.

7 Muffins, 3 Students

Work on $f(7,3)$ in groups in breakout rooms.
7 Muffins, 3 Students.
Get upper and lower bounds that match!

7 Muffins, 3 Students: How to think about it

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3. 7 muffins, each one cut in two 2 pieces, so 14 pieces total.

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7. Can we show a protocol that gives $f(7,3) \geq \frac{7}{15}$.

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8. We tried. We failed. Darn :-(

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Now what?

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5. That student must get a piece $\geq \frac{7}{3} \times \frac{1}{4} = \frac{7}{12}$.
6. That piece came from a muffin. Other piece is $\leq 1 - \frac{7}{12} = \frac{5}{12}$.

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7. Great! We know $f(7,3) \le \frac{5}{12}$.
8. Can we show a protocol that gives $f(7,3) \ge \frac{5}{12}$?

7 Muffins, 3 Students: How to think about protocol

Want $f(7,3) \geq \frac{5}{12}$.

7 Muffins, 3 Students: How to think about protocol

Want $f(7,3) \geq \frac{5}{12}$.
Will be cutting some muffins $(\frac{5}{12}, \frac{7}{12})$.

7 Muffins, 3 Students: How to think about protocol

Want $f(7,3) \geq \frac{5}{12}$.
Will be cutting some muffins $(\frac{5}{12}, \frac{7}{12})$.
Can also cut some muffins $(\frac{6}{12}, \frac{6}{12})$.

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Want $f(7,3) \geq \frac{5}{12}$.
Will be cutting some muffins $(\frac{5}{12}, \frac{7}{12})$.
Can also cut some muffins $(\frac{6}{12}, \frac{6}{12})$.
Need to know what combos of $\frac{5}{12}, \frac{6}{12}, \frac{7}{12}$ add to $\frac{7}{3} = \frac{28}{12}$.

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Want $f(7,3) \geq \frac{5}{12}$.
Will be cutting some muffins $(\frac{5}{12}, \frac{7}{12})$.
Can also cut some muffins $(\frac{6}{12}, \frac{6}{12})$.
Need to know what combos of $\frac{5}{12}, \frac{6}{12}, \frac{7}{12}$ add to $\frac{7}{3} = \frac{28}{12}$.
Need to know what combos of $5, 6, 7$ add to 28.

$7 + 7 + 7 + 7 = 28$

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Will be cutting some muffins $(\frac{5}{12}, \frac{7}{12})$.
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Need to know what combos of $5, 6, 7$ add to 28.

$7 + 7 + 7 + 7 = 28$

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1. Cut 4 muffins $(\frac{5}{12}, \frac{7}{12})$.

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Need to know what combos of $5, 6, 7$ add to 28.
$7 + 7 + 7 + 7 = 28$
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1. Cut 4 muffins $(\frac{5}{12}, \frac{7}{12})$.
2. Cut 3 muffins $(\frac{6}{12}, \frac{6}{12})$.

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Will be cutting some muffins $(\frac{5}{12}, \frac{7}{12})$.
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$7 + 7 + 7 + 7 = 28$
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1. Cut 4 muffins $(\frac{5}{12}, \frac{7}{12})$.
2. Cut 3 muffins $(\frac{6}{12}, \frac{6}{12})$.
3. Give 1 student 4 pieces of size $\frac{7}{12}$.

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Want $f(7,3) \geq \frac{5}{12}$.
Will be cutting some muffins $(\frac{5}{12}, \frac{7}{12})$.
Can also cut some muffins $(\frac{6}{12}, \frac{6}{12})$.
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1. Cut 4 muffins $(\frac{5}{12}, \frac{7}{12})$.
2. Cut 3 muffins $(\frac{6}{12}, \frac{6}{12})$.
3. Give 1 student 4 pieces of size $\frac{7}{12}$.
4. Give 2 students 2 pieces of size $\frac{5}{12}$ and 3 pieces of size $\frac{6}{12}$.

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1. If a muffin is uncut, can cut it in two.
2. If a muffin is cut in ≥ 3 pieces then some piece $\leq \frac{1}{3}$. Unlikely that thats a good idea.
3. 8 muffins, each one cut in two 2 pieces, so 16 pieces total.

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4. 3 students, so some student gets $\geq \lceil \frac{16}{3} \rceil = 6$ pieces. That student must get a piece $\leq \frac{8}{3} \times \frac{1}{6} = \frac{4}{9}$.

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5. 3 students, so some student gets $\leq \lfloor \frac{16}{3} \rfloor = 5$ pieces. That student must get a piece $\geq \frac{8}{3} \times \frac{1}{5} = \frac{8}{15}$. So there is some piece of size $\leq 1 - \frac{8}{15} = \frac{7}{15}$.

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6. Great! We know $f(8,3) \leq \min\{\frac{4}{9}, \frac{7}{15}\} = \frac{4}{9}$.

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6. Great! We know $f(8,3) \le \min\{\frac{4}{9}, \frac{7}{15}\} = \frac{4}{9}$.
7. Can we show a protocol that gives $f(8,3) \ge \frac{4}{9}$?

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Want $f(8,3) \geq \frac{4}{9}$.

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Want $f(8,3) \geq \frac{4}{9}$.
Will be cutting some muffins $(\frac{4}{9}, \frac{5}{9})$.

8 Muffins, 3 Students: How to think about protocol

Want $f(8,3) \geq \frac{4}{9}$.
Will be cutting some muffins $(\frac{4}{9}, \frac{5}{9})$.
$\frac{1}{2}$ was helpful last time so lets also include $\frac{4.5}{9}$.
Need to know what combos of $\frac{4}{9}, \frac{4.5}{9}, \frac{5}{9}$ add to $\frac{8}{3} = \frac{24}{9}$.

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Want $f(8,3) \geq \frac{4}{9}$.
Will be cutting some muffins $(\frac{4}{9}, \frac{5}{9})$.
$\frac{1}{2}$ was helpful last time so lets also include $\frac{4.5}{9}$.
Need to know what combos of $\frac{4}{9}, \frac{4.5}{9}, \frac{5}{9}$ add to $\frac{8}{3} = \frac{24}{9}$.
Need to know what combos of $4, 4.5, 5$ add to 24

$4+4+4+4+4+4=24$

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Will be cutting some muffins $(\frac{4}{9}, \frac{5}{9})$.
$\frac{1}{2}$ was helpful last time so lets also include $\frac{4.5}{9}$.
Need to know what combos of $\frac{4}{9}, \frac{4.5}{9}, \frac{5}{9}$ add to $\frac{8}{3} = \frac{24}{9}$.
Need to know what combos of $4, 4.5, 5$ add to 24
$4+4+4+4+4+4 = 24$
$4.5+4.5+5+5+5 = 24$

1. Cut 6 muffins $(\frac{4}{9}, \frac{5}{9})$.
2. Cut 2 muffins $(\frac{4.5}{9}, \frac{4.5}{9})$.
3. Give 1 student six $\frac{4}{9}$ pieces.
4. Give 2 students two $\frac{4.5}{9}$ pieces and four $\frac{5}{9}$ pieces.

$f(3,5) \geq ?$

Clearly $f(3,5) \geq \frac{1}{5}$.

Can we get $f(3,5) > \frac{1}{5}$?

Work on it with your neighbor

$f(3,5) \geq \frac{1}{4}$

$f(3,5) \geq \frac{1}{4}$

1. Divide 2 muffin $[\frac{6}{20}, \frac{7}{20}, \frac{7}{20}]$
2. Divide 1 muffin $[\frac{5}{20}, \frac{5}{20}, \frac{5}{20}, \frac{5}{20}]$
3. Give 4 students $(\frac{5}{20}, \frac{7}{20})$
4. Give 1 students $(\frac{6}{20}, \frac{6}{20})$

$f(3,5) \geq \frac{1}{4}$

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1. Divide 2 muffin $[\frac{6}{20}, \frac{7}{20}, \frac{7}{20}]$
2. Divide 1 muffin $[\frac{5}{20}, \frac{5}{20}, \frac{5}{20}, \frac{5}{20}]$
3. Give 4 students $(\frac{5}{20}, \frac{7}{20})$
4. Give 1 students $(\frac{6}{20}, \frac{6}{20})$

Can we do better?

Work on it with your neighbor

3 Muffins, 5 People–Can't Do Better Than $\frac{1}{4}$

NO WE CAN'T!

There is a procedure for 3 muffins,5 students where each student gets $\frac{3}{5}$ muffins, smallest piece N. We want $N \leq \frac{1}{4}$.

3 Muffins, 5 People–Can't Do Better Than $\frac{1}{4}$

NO WE CAN'T!

There is a procedure for 3 muffins,5 students where each student gets $\frac{3}{5}$ muffins, smallest piece N. We want $N \le \frac{1}{4}$.

Case 0: Alice gets 1 piece of size $\frac{3}{5}$. Look at the rest of that muffin which totals to $\frac{2}{5}$. (1) That piece is cut. Have piece $\le \frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$, OR (2) That piece uncut. So someone gets a $\frac{2}{5}$-piece. Must also get a $\frac{1}{5}$ piece.
(**Henceforth:** All people get ≥ 2 pieces.)

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Case 1: Alice gets ≥ 3 pieces. Then $N \le \frac{3}{5} \times \frac{1}{3} = \frac{1}{5}$.
(**Henceforth:** Everyone gets 2 pieces.)

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(**Henceforth:** All people get ≥ 2 pieces.)

Case 1: Alice gets ≥ 3 pieces. Then $N \le \frac{3}{5} \times \frac{1}{3} = \frac{1}{5}$.
(**Henceforth:** Everyone gets 2 pieces.)

Case 2: Everyone gets 2 pieces. 10 pieces, 3 muffins:
Some muffin gets ≥ 4 pieces. So some piece is $\le \frac{1}{4}$.

$f(3,5)$ and $f(5,3)$

$f(5,3) \geq \frac{5}{12}$

1. Divide 4 muffins $[\frac{5}{12}, \frac{7}{12}]$
2. Divide 1 muffin $[\frac{6}{12}, \frac{6}{12}]$
3. Give 2 students $(\frac{6}{12}, \frac{7}{12}, \frac{7}{12})$
4. Give 1 students $(\frac{5}{12}, \frac{5}{12}, \frac{5}{12}, \frac{5}{12})$

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1. Divide 4 muffins $[\frac{5}{12}, \frac{7}{12}]$
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4. Give 1 students $(\frac{5}{12}, \frac{5}{12}, \frac{5}{12}, \frac{5}{12})$

$f(3,5) \geq \frac{1}{4}$

1. Divide 2 muffin $[\frac{6}{20}, \frac{7}{20}, \frac{7}{20}]$

$f(3,5)$ and $f(5,3)$

$f(5,3) \geq \frac{5}{12}$

1. Divide 4 muffins $[\frac{5}{12}, \frac{7}{12}]$
2. Divide 1 muffin $[\frac{6}{12}, \frac{6}{12}]$
3. Give 2 students $(\frac{6}{12}, \frac{7}{12}, \frac{7}{12})$
4. Give 1 students $(\frac{5}{12}, \frac{5}{12}, \frac{5}{12}, \frac{5}{12})$

$f(3,5) \geq \frac{1}{4}$

1. Divide 2 muffin $[\frac{6}{20}, \frac{7}{20}, \frac{7}{20}]$
2. Divide 1 muffin $[\frac{5}{20}, \frac{5}{20}, \frac{5}{20}, \frac{5}{20}]$
3. Give 4 students $(\frac{5}{20}, \frac{7}{20})$
4. Give 1 students $(\frac{6}{20}, \frac{6}{20})$

$f(3,5)$ and $f(5,3)$

$f(5,3) \geq \frac{5}{12}$

1. Divide 4 muffins $[\frac{5}{12}, \frac{7}{12}]$
2. Divide 1 muffin $[\frac{6}{12}, \frac{6}{12}]$
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$f(3,5) \geq \frac{1}{4}$

1. Divide 2 muffin $[\frac{6}{20}, \frac{7}{20}, \frac{7}{20}]$
2. Divide 1 muffin $[\frac{5}{20}, \frac{5}{20}, \frac{5}{20}, \frac{5}{20}]$
3. Give 4 students $(\frac{5}{20}, \frac{7}{20})$
4. Give 1 students $(\frac{6}{20}, \frac{6}{20})$

$f(3,5)$ proc is $f(5,3)$ proc but swap Divide/Give and mult by $3/5$.

$f(3,5)$ and $f(5,3)$

$f(5,3) \ge \frac{5}{12}$

1. Divide 4 muffins $[\frac{5}{12}, \frac{7}{12}]$
2. Divide 1 muffin $[\frac{6}{12}, \frac{6}{12}]$
3. Give 2 students $(\frac{6}{12}, \frac{7}{12}, \frac{7}{12})$
4. Give 1 students $(\frac{5}{12}, \frac{5}{12}, \frac{5}{12}, \frac{5}{12})$

$f(3,5) \ge \frac{1}{4}$

1. Divide 2 muffin $[\frac{6}{20}, \frac{7}{20}, \frac{7}{20}]$
2. Divide 1 muffin $[\frac{5}{20}, \frac{5}{20}, \frac{5}{20}, \frac{5}{20}]$
3. Give 4 students $(\frac{5}{20}, \frac{7}{20})$
4. Give 1 students $(\frac{6}{20}, \frac{6}{20})$

$f(3,5)$ proc is $f(5,3)$ proc but swap Divide/Give and mult by 3/5.
Duality Theorem: $f(m,s) = \frac{m}{s} f(s,m)$.

Conventions

We know and use the following:

1. By duality can assume $m > s$
2. If s divides m then $f(m,s) = 1$ so assume s does not divide m.
3. All muffins are cut in ≥ 2 pcs. Replace uncut muff with 2 $\frac{1}{2}$'s
4. If assuming $f(m,s) > \alpha > \frac{1}{3}$, assume all muffin in ≤ 2 pcs.
5. $f(m,s) > \alpha > \frac{1}{3}$, so exactly 2 pcs, is common case.

We do not know this but still use: $f(m,s)$ only depends on $\frac{m}{s}$.
All of our techniques that hold for (m,s) hold for (Am, As).
For particular numbers, we only look at m, s rel prime.

FC Thm Generalizes $f(5,3) \le \frac{5}{12}$

$$f(m,s) \le FC(m,s) = \max\left\{\frac{1}{3}, \min\left\{\frac{m}{s\lceil 2m/s\rceil}, 1 - \frac{m}{s\lfloor 2m/s\rfloor}\right\}\right\}.$$

Case 0: Some muffin is uncut. Cut it $(\frac{1}{2}, \frac{1}{2})$ and give both halves to whoever got the uncut muffin, so reduces to other cases.

Case 1: Some muffin is cut into ≥ 3 pieces. Some piece $\le \frac{1}{3}$.

Case 2: Every muffin is cut into 2 pieces, so $2m$ pieces.

Someone gets $\ge \lceil \frac{2m}{s} \rceil$ pieces. $\exists$ piece $\le \frac{m}{s} \times \frac{1}{\lceil 2m/s\rceil} = \frac{m}{s\lceil 2m/s\rceil}$.

Someone gets $\le \lfloor \frac{2m}{s} \rfloor$ pieces. $\exists$ piece $\ge \frac{m}{s}\frac{1}{\lfloor 2m/s\rfloor} = \frac{m}{s\lfloor 2m/s\rfloor}$.

The other piece from that muffin is of size $\le 1 - \frac{m}{s\lfloor 2m/s\rfloor}$.

THREE Students

CLEVERNESS, COMP PROGS for the procedure.

FC Theorem for optimality.

$f(1,3) = \frac{1}{3}$

$f(3k,3) = 1.$

$f(3k+1,3) = \frac{3k-1}{6k},\ k \ge 1.$

$f(3k+2,3) = \frac{3k+2}{6k+6}.$

Note: A Mod 3 Pattern.
Theorem: For all $m \ge 3$, $f(m,3) = \mathrm{FC}(m,3)$.

FOUR Students

CLEVERNESS, COMP PROGS for procedures.

FC Theorem for optimality.

$f(4k, 4) = 1$ (easy)

$f(1, 4) = \frac{1}{4}$ (easy)

$f(4k+1, 4) = \frac{4k-1}{8k}$, $k \geq 1$.

$f(4k+2, 4) = \frac{1}{2}$.

$f(4k+3, 4) = \frac{4k+1}{8k+4}$.

Note: A Mod 4 Pattern.
Theorem: For all $m \geq 4$, $f(m, 4) = \mathrm{FC}(m, 4)$.
FC-Conjecture: For all m, s with $m \geq s$, $f(m, s) = \mathrm{FC}(m, s)$.

FIVE Students

CLEVERNESS, COMP PROGS for procedures.

FC Theorem for optimality.

For $k \geq 1$, $f(5k, 5) = 1$.

For $k = 1$ and $k \geq 3$, $f(5k+1, 5) = \frac{5k+1}{10k+5}$. $f(11, 5)$?

For $k \geq 2$, $f(5k+2, 5) = \frac{5k-2}{10k}$. $f(7, 5) = \mathrm{FC}(7, 5) = \frac{1}{3}$

For $k \geq 1$, $f(5k+3, 5) = \frac{5k+3}{10k+10}$

For $k \geq 1$, $f(5k+4, 5) = \frac{5k+1}{10k+5}$
Note: A Mod 5 Pattern.
Theorem: For all $m \geq 5$ **except m=11**, $f(m, 5) = \mathrm{FC}(m, 5)$.

What About FIVE students, ELEVEN muffins?

$$f(11,5) \leq \max\left\{\frac{1}{3}, \min\left\{\frac{11}{5\lceil 22/5\rceil}, 1-\frac{11}{5\lfloor 22/5\rfloor}\right\}\right\} = \frac{11}{25}.$$

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We tried to find a protocol to divide 11 muffins for 5 people, each gets $\frac{11}{5}$, and smallest piece is size $\frac{11}{25} = 0.44$.

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We tried to find a protocol to divide 11 muffins for 5 people, each gets $\frac{11}{5}$, and smallest piece is size $\frac{11}{25} = 0.44$.
We found a protocol with smallest piece $\frac{13}{30} = 0.4333\ldots$.

1. Divide 1 muffin $(\frac{15}{30}, \frac{15}{30})$.
2. Divide 2 muffins $(\frac{14}{30}, \frac{16}{30})$.
3. Divide 8 muffins $(\frac{13}{30}, \frac{17}{30})$.
4. Give 2 students $[\frac{13}{30}, \frac{13}{30}, \frac{13}{30}, \frac{13}{30}, \frac{14}{30}]$
5. Give 1 students $[\frac{16}{30}, \frac{16}{30}, \frac{17}{30}, \frac{17}{30}]$
6. Give 2 students $[\frac{15}{30}, \frac{17}{30}, \frac{17}{30}, \frac{17}{30}]$

So Now What?

We have:

$$\frac{13}{30} \le f(11,5) \le \frac{11}{25} \qquad \text{Diff}= 0.006666\ldots$$

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$$\frac{13}{30} \le f(11,5) \le \frac{11}{25} \qquad \text{Diff} = 0.006666\ldots$$

Options:

1. $f(11,5) = \frac{11}{25}$. Need to find procedure.
2. $f(11,5) = \frac{13}{30}$. Need to find new technique for upper bounds.
3. $f(11,5)$ in between. Need to find both.
4. $f(11,5)$ unknown to science!

Vote

So Now What?

We have:

$$\frac{13}{30} \leq f(11,5) \leq \frac{11}{25} \quad \text{Diff}= 0.006666\ldots$$

Options:

1. $f(11,5) = \frac{11}{25}$. Need to find procedure.
2. $f(11,5) = \frac{13}{30}$. Need to find new technique for upper bounds.
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4. $f(11,5)$ unknown to science!

Vote WE SHOW: $\mathbf{f(11,5) = \frac{13}{30}}$. **Exciting** new technique!

Terminology: Buddy

Assume that in some protocol every muffin is cut into two pieces.

Let x be a piece from muffin M.
The *other piece* from muffin M is the *buddy of* x.

Note that the buddy of x is of size

$$1 - x.$$

$f(11,5)=\frac{13}{30}$, Easy Case Based on Muffins

There is a procedure for 11 muffins, 5 students where each student gets $\frac{11}{5}$ muffins, smallest piece N. We want $N \le \frac{13}{30}$.

Case 0: Some muffin is uncut. Cut it $(\frac{1}{2},\frac{1}{2})$ and give both halves to whoever got the uncut muffin. Reduces to other cases.

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Case 0: Some muffin is uncut. Cut it $(\frac{1}{2},\frac{1}{2})$ and give both halves to whoever got the uncut muffin. Reduces to other cases.

Case 1: Some muffin is cut into ≥ 3 pieces. $N \le \frac{1}{3} < \frac{13}{30}$.

(**Negation of Case 0 and Case 1:** All muffins cut into 2 pieces.)

$f(11,5) = \frac{13}{30}$, Easy Case Based on Students

Case 2: Some student gets ≥ 6 pieces.

$$N \leq \frac{11}{5} \times \frac{1}{6} = \frac{11}{30} < \frac{13}{30}.$$

$f(11,5) = \frac{13}{30}$, Easy Case Based on Students

Case 2: Some student gets ≥ 6 pieces.

$$N \leq \frac{11}{5} \times \frac{1}{6} = \frac{11}{30} < \frac{13}{30}.$$

Case 3: Some student gets ≤ 3 pieces.
One of the pieces is

$$\geq \frac{11}{5} \times \frac{1}{3} = \frac{11}{15}.$$

Look at the muffin it came from to find a piece that is

$$\leq 1 - \frac{11}{15} = \frac{4}{15} < \frac{13}{30}.$$

(**Negation of Cases 2 and 3:** Every student gets 4 or 5 pieces.)

$f(11,5) = \frac{13}{30}$, Fun Cases

Case 4: Every muffin is cut in 2 pieces, every student gets 4 or 5 pieces. Number of pieces: 22. Note ≤ 11 pieces are $> \frac{1}{2}$.

- s_4 is number of students who get 4 pieces
- s_5 is number of students who get 5 pieces

$$\begin{aligned} 4s_4 + 5s_5 &= 22 \\ s_4 + s_5 &= 5 \end{aligned}$$

$s_4 = 3$: There are 3 students who have 4 shares.
$s_5 = 2$: There are 2 students who have 5 shares.

We call a share that goes to a person who gets 4 shares a **4-share**.
We call a share that goes to a person who gets 5 shares a **5-share**.

$f(11,5) = \frac{13}{30}$, Fun Cases

Case 4.1: Some 4-share is $\le \frac{1}{2}$.
Alice gets w, x, y, z and $w \le \frac{1}{2}$.
Since $w + x + y + z = \frac{11}{5}$ and $w \le \frac{1}{2}$

$$x + y + z \ge \frac{11}{5} - \frac{1}{2} = \frac{17}{10}$$

Let x be the largest of x, y, z

$$x \ge \frac{17}{10} \times \frac{1}{3} = \frac{17}{30}$$

Look at **buddy** of x.

$$B(x) \le 1 - x = 1 - \frac{17}{30} = \frac{13}{30}$$

GREAT! This is where $\frac{13}{30}$ comes from!

$f(11,5) = \frac{13}{30}$, Fun Cases

Case 4.2: All 4-shares are $> \frac{1}{2}$. There are $4s_4 = 12$ 4-shares. There are ≥ 12 pieces $> \frac{1}{2}$. Can't occur.

Later Results by Other People

1. In Fall 2018 Scott Huddleston had code for an algorithm that, on input m, s, found $f(m, s)$ and the procedure REALLY FAST.
2. Jacob and Erik Understand WHAT his algorithm does and Jacob coded it up to make sure he understood it. Jacob's code is also REALLY FAST.
3. Neither Scott, Bill, Jacob, or Erik had a proof that Scott's algorithm was fast (poly in m, s).
4. Richard Chatwin independently came up with the same algorithm; however, he also has a proof that it works. Its on arXiv.
5. One corollary of the work: $f(m, s)$ only depends on m/s.

I meet Alan Frank!

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- He does a Bike-For-Food Charity. I asked him if I should give \$40.00 a year OR my Royalties. He chose the \$40.00.
  First Year Royalties: \$41.00
  Second Year Royalties: Don't know yet but will be $<$ \$40.00.