The Muffin Problem

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How it Began

A Recreational Math Conference
(Gathering for Gardner)
May 2016

I found a pamphlet:

The Julia Robinson Mathematics Festival:
A Sample of Mathematical Puzzles
Compiled by Nancy Blachman

which had this problem, proposed by Alan Frank:

How can you divide and distribute 5 muffins to 3 students so that every student gets \(\frac{5}{3}\) where nobody gets a tiny sliver?
## Five Muffins, Three Students, Proc by Picture

<table>
<thead>
<tr>
<th>Person</th>
<th>Color</th>
<th>What they Get</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alice</td>
<td>RED</td>
<td>$1 + \frac{2}{3} = \frac{5}{3}$</td>
</tr>
<tr>
<td>Bob</td>
<td>BLUE</td>
<td>$1 + \frac{2}{3} = \frac{5}{3}$</td>
</tr>
<tr>
<td>Carol</td>
<td>GREEN</td>
<td>$1 + \frac{1}{3} + \frac{1}{3} = \frac{5}{3}$</td>
</tr>
</tbody>
</table>

Smallest Piece: $\frac{1}{3}$
Can We Do Better?

The smallest piece in the above solution is $\frac{1}{3}$.

**Is there a procedure with a larger smallest piece?**

**Work on it with your neighbor**
**Five Muffins, Three People–Proc by Picture**

**YES WE CAN!**

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<td>RED</td>
<td>$\frac{6}{12} + \frac{7}{12} + \frac{7}{12}$</td>
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<tr>
<td>Bob</td>
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<td>$\frac{5}{12} + \frac{5}{12} + \frac{5}{12} + \frac{5}{12}$</td>
</tr>
</tbody>
</table>

**Smallest Piece:** $\frac{5}{12}$
Can We Do Better?

The smallest piece in the above solution is $\frac{5}{12}$. Is there a procedure with a larger smallest piece? Work on it with your neighbor.
NO WE CAN’T!

There is a procedure for 5 muffins, 3 students where each student gets $\frac{5}{3}$ muffins, smallest piece $N$. We want $N \leq \frac{5}{12}$. 
NO WE CAN’T!

There is a procedure for 5 muffins, 3 students where each student gets $\frac{5}{3}$ muffins, smallest piece $N$. We want $N \leq \frac{5}{12}$.

**Case 0:** Some muffin is uncut. Cut it ($\frac{1}{2}, \frac{1}{2}$) and give both $\frac{1}{2}$-sized pieces to whoever got the uncut muffin. (Note $\frac{1}{2} > \frac{5}{12}$.) Reduces to other cases.
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(Henceforth: All muffins are cut into $\geq 2$ pieces.)
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**Case 1:** Some muffin is cut into \( \geq 3 \) pieces. Then \( N \leq \frac{1}{3} < \frac{5}{12} \).
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(Henceforth: All muffins are cut into $\geq 2$ pieces.)

Case 1: Some muffin is cut into $\geq 3$ pieces. Then $N \leq \frac{1}{3} < \frac{5}{12}$.
(Henceforth: All muffins are cut into 2 pieces.)

Case 2: All muffins are cut into 2 pieces. 10 pieces, 3 students: Someone gets $\geq 4$ pieces. He has some piece

$$\leq \frac{5}{3} \times \frac{1}{4} = \frac{5}{12}$$  Great to see $\frac{5}{12}$
What Happened Next?
What Happened Next?

Yada Yada Yada- in 2020:
What Happened Next?

Yada Yada Yada- in 2020:

**MATHEMATICAL MUFFIN MORSELS: NOBODY WANTS A SMALL PIECE**

William Gasarch, Erik Metz, Jacob Prinz, Daniel Smolyak

*University of Maryland, USA*

In this book we consider THE MUFFIN PROBLEM: what is the best way to divide up m muffins for s students so that everyone gets m/s muffins, with the smallest pieces maximized.

This problem takes us through much mathematics of interest, for example, combinatorics and optimization theory.

228pp
978-981-121-597-1(pbkw) US$28 / £25 / SGD41
978-981-121-517-9 US$58 / £50 / SGD86
978-981-121-519-3(mbook) US$22 / £20 / SGD33

Is there a way to divide five muffins for three students so that everyone gets 5/3, and all pieces are larger than 1/3? Spoiler alert: Yes!

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https://doi.org/10.1142/11689
General Problem

$f(m, s)$ be the smallest piece in the best procedure (best in that the smallest piece is maximized) to divide $m$ muffins among $s$ students so that everyone gets $\frac{m}{s}$.

We have shown $f(5, 3) = \frac{5}{12}$ here.
We Only Deal with $m > s$

**Duality Theorem:** $f(m, s) = \frac{m}{s} f(s, m)$.
Hence we will just look at $m > s$. 
Floor-Ceiling Thm Generalizes $f(5, 3) \leq \frac{5}{12}$

$$f(m, s) \leq \text{FC}(m, s) = \max \left\{ \frac{1}{3}, \min \left\{ \frac{m}{s \lceil 2m/s \rceil}, 1 - \frac{m}{s \lfloor 2m/s \rfloor} \right\} \right\}.$$
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**Case 2:** Every muffin is cut into 2 pieces, so $2m$ pieces.
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**Someone** gets $\geq \left\lceil \frac{2m}{s} \right\rceil$ pieces. $\exists$ piece $\leq \frac{m}{s} \times \frac{1}{\lfloor 2m/s \rfloor} = \frac{m}{s \lfloor 2m/s \rfloor}$. 
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The other piece from that muffin is of size $\leq 1 - \frac{m}{s} \left\lfloor \frac{2m}{s} \right\rfloor$. 
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The other piece from that muffin is of size $\leq 1 - \frac{m}{s \lceil 2m/s \rceil}$. 
Give $m, s$:

$$\text{FC}(m, s) = \max \left\{ \frac{1}{3}, \min \left\{ \frac{m}{s \lfloor 2m/s \rfloor}, 1 - \frac{m}{s \lfloor 2m/s \rfloor} \right\} \right\}$$

Gives an upper bound. So we know

$$(\forall m, s)[f(m, s) \leq \text{FC}(m, s)].$$

Is the following true?

$$(\forall m, s)[f(m, s) = \text{FC}(m, s)]$$
Give $m, s$:

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Gives an upper bound. So we know

$$(\forall m, s) [f(m, s) \leq FC(m, s)]$$.

Is the following true?

$$(\forall m, s) [f(m, s) = FC(m, s)]$$

**No:** If so my book would be about 20 pages.
TWO, THREE, FOUR, FIVE Students

\textbf{Thm}

1. For all $m \geq 3$, $f(m, 2) = FC(m, 2)$.
2. For all $m \geq 4$, $f(m, 3) = FC(m, 3)$.
3. For all $m \geq 5$, $f(m, 4) = FC(m, 4)$. 
Thm For all $m \geq 6$, $f(m, 5) = FC(m, 5)$
Thm For all $m \geq 6$, $f(m, 5) = FC(m, 5)$
Except $m = 11$. What!
Thm For all $m \geq 6$, $f(m, 5) = FC(m, 5)$
Except $m = 11$. What!

1. We have a procedure which shows $f(11, 5) \geq \frac{13}{30}$.
2. $f(11, 5) \leq \max\left\{\frac{1}{3}, \min\left\{\frac{11}{5\lceil22/5\rceil}, 1 - \frac{11}{5\lfloor22/5\rfloor}\right\}\right\} = \frac{11}{25}$. 
**Thm** For all $m \geq 6$, $f(m, 5) = \text{FC}(m, 5)$

Except $m = 11$. What!

1. We have a procedure which shows $f(11, 5) \geq \frac{13}{30}$.
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So

$$\frac{13}{30} \leq f(11, 5) \leq \frac{11}{25} \quad \text{Diff} = 0.006666\ldots$$
**Thm** For all $m \geq 6$, $f(m, 5) = FC(m, 5)$

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So

$$\frac{13}{30} \leq f(11, 5) \leq \frac{11}{25} \quad \text{Diff}= 0.006666\ldots$$

**WE SHOW** $f(11, 5) = \frac{13}{30}$. **Exciting** new technique!
Assume that in some protocol every muffin is cut into two pieces.

Let \( x \) be a piece from muffin \( M \).
The other piece from muffin \( M \) is the buddy of \( x \).

Note that the buddy of \( x \) is of size

\[ 1 - x. \]
There is a procedure for 11 muffins, 5 students where each student gets \( \frac{11}{5} \) muffins, smallest piece \( N \). We want \( N \leq \frac{13}{30} \).

**Case 0:** Some muffin is uncut. Cut it \( \left(\frac{1}{2}, \frac{1}{2}\right) \) and give both halves to whoever got the uncut muffin. Reduces to other cases.
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**Case 1:** Some muffin is cut into $\geq 3$ pieces. $N \leq \frac{1}{3} < \frac{13}{30}$. 

$f(11, 5) = \frac{13}{30}$, Easy Case Based on Muffins
There is a procedure for 11 muffins, 5 students where each student gets \( \frac{11}{5} \) muffins, smallest piece \( N \). We want \( N \leq \frac{13}{30} \).

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**Case 1:** Some muffin is cut into \( \geq 3 \) pieces. \( N \leq \frac{1}{3} < \frac{13}{30} \).

(*Negation of Case 0 and Case 1: All muffins cut into 2 pieces.*)
$f(11, 5) = \frac{13}{30}$, Easy Case Based on Students

**Case 2:** Some student gets $\geq 6$ pieces.

$$N \leq \frac{11}{5} \times \frac{1}{6} = \frac{11}{30} < \frac{13}{30}.$$ 

**Case 3:** Some student gets $\leq 3$ pieces.

One of the pieces is

$$\geq \frac{11}{5} \times \frac{1}{3} = \frac{11}{15}.$$
$f(11, 5) = \frac{13}{30}$, Easy Case Based on Students

**Case 2:** Some student gets $\geq 6$ pieces.

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That pieces *buddy* is of size:

$$\leq 1 - \frac{11}{15} = \frac{4}{15} < \frac{13}{30}.$$
\[ f(11, 5) = \frac{13}{30}, \text{ Easy Case Based on Students} \]

**Case 2:** Some student gets \( \geq 6 \) pieces.

\[
N \leq \frac{11}{5} \times \frac{1}{6} = \frac{11}{30} < \frac{13}{30}.
\]

**Case 3:** Some student gets \( \leq 3 \) pieces.

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That pieces **buddy** is of size:

\[
\leq 1 - \frac{11}{15} = \frac{4}{15} < \frac{13}{30}.
\]

**(Negation of Cases 2 and 3:** Every student gets 4 or 5 pieces.)
$f(11, 5) = \frac{13}{30}$, Fun Cases

**Case 4:** Every muffin is cut in 2 pieces, every student gets 4 or 5 pieces. Number of pieces: 22. Note $\leq 11$ pieces are $> \frac{1}{2}$.

$s_4$ is number of students who get 4 pieces

$s_5$ is number of students who get 5 pieces

$s_4 + 5s_5 = 22$

$s_4 = 3$: There are 3 students who have 4 shares.

$s_5 = 2$: There are 2 students who have 5 shares.

We call a share that goes to a person who gets 4 shares a 4-share.

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Case 4: Every muffin is cut in 2 pieces, every student gets 4 or 5 pieces. Number of pieces: 22. Note $\leq 11$ pieces are $> \frac{1}{2}$.

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\[
4s_4 + 5s_5 = 22 \\
s_4 + s_5 = 5
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$f(11, 5) = \frac{13}{30}$, Fun Cases

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- $s_5$ is number of students who get 5 pieces

\[4s_4 + 5s_5 = 22\]
\[s_4 + s_5 = 5\]

$s_4 = 3$: There are 3 students who have 4 shares.
$s_5 = 2$: There are 2 students who have 5 shares.

We call a share that goes to a person who gets 4 shares a **4-share**.
We call a share that goes to a person who gets 5 shares a **5-share**.
\( f(11, 5) = \frac{13}{30}, \text{ Fun Cases} \)

**Case 4.1:** Some 4-share is \( \leq \frac{1}{2} \).
$f(11, 5) = \frac{13}{30}$, Fun Cases

**Case 4.1:** Some 4-share is $\leq \frac{1}{2}$.

Alice gets $w, x, y, z$ and $w \leq \frac{1}{2}$.

Since $w + x + y + z = \frac{11}{5}$ and $w \leq \frac{1}{2}$

$$x + y + z \geq \frac{11}{5} - \frac{1}{2} = \frac{17}{10}$$
Case 4.1: Some 4-share is $\leq \frac{1}{2}$.

Alice gets $w, x, y, z$ and $w \leq \frac{1}{2}$.

Since $w + x + y + z = \frac{11}{5}$ and $w \leq \frac{1}{2}$

$$x + y + z \geq \frac{11}{5} - \frac{1}{2} = \frac{17}{10}$$

Let $x$ be the largest of $x, y, z$

$$x \geq \frac{17}{10} \times \frac{1}{3} = \frac{17}{30}$$
\( f(11, 5) = \frac{13}{30} \), Fun Cases

**Case 4.1:** Some 4-share is \( \leq \frac{1}{2} \).

Alice gets \( w, x, y, z \) and \( w \leq \frac{1}{2} \).

Since \( w + x + y + z = \frac{11}{5} \) and \( w \leq \frac{1}{2} \),

\[
x + y + z \geq \frac{11}{5} - \frac{1}{2} = \frac{17}{10}
\]

Let \( x \) be the largest of \( x, y, z \)

\[
x \geq \frac{17}{10} \times \frac{1}{3} = \frac{17}{30}
\]

The **buddy** of \( x \) is of size

\[
\leq 1 - x = 1 - \frac{17}{30} = \frac{13}{30}
\]
Case 4.1: Some 4-share is $\leq \frac{1}{2}$. Alice gets $w, x, y, z$ and $w \leq \frac{1}{2}$.
Since $w + x + y + z = \frac{11}{5}$ and $w \leq \frac{1}{2}$

$$x + y + z \geq \frac{11}{5} - \frac{1}{2} = \frac{17}{10}$$

Let $x$ be the largest of $x, y, z$

$$x \geq \frac{17}{10} \times \frac{1}{3} = \frac{17}{30}$$

The buddy of $x$ is of size

$$\leq 1 - x = 1 - \frac{17}{30} = \frac{13}{30}$$

GREAT! This is where $\frac{13}{30}$ comes from!
$f(11, 5) = \frac{13}{30}$, Fun Cases

**Case 4.2:** All 4-shares are $> \frac{1}{2}$. There are $4s_4 = 12$ 4-shares. There are $\geq 12$ pieces $> \frac{1}{2}$. Can’t occur.
This Kept Happening!

1. We Generalized the method for $f(11, 5)$ and called it HALF.
2. We use FC and HALF to solve MANY problems.
3. We found obstacles and found methods to overcome them.
4. Next slide tells you the names of the methods and how often they worked.
All of Our Methods

Let

\[ A = \{(m, s) \mid 2 \leq s \leq 100 \text{ and } s < m \leq 110 \text{ and } m, s \text{ rel prime}\} \]

There are 3520 pairs \((m, s)\) in \(A\). We solved all of them!

- For 2301 of them \(f(m, s) = FC(m, s)\). That is \(\sim 65.37\%\).
- For 329 of them \(f(m, s) = \text{HALF}(m, s)\). That is \(\sim 9.35\%\).
- For 186 of them \(f(m, s) = \text{INT}(m, s)\). That is \(\sim 5.28\%\).
- For 111 of them \(f(m, s) = \text{MID}(m, s)\). That is \(\sim 3.15\%\).
- For 240 of them \(f(m, s) = \text{EBM}(m, s)\). That is \(\sim 6.28\%\).
- For 89 of them \(f(m, s) = \text{HBM}(m, s)\). That is \(\sim 2.53\%\).
- For 250 of them \(f(m, s) = \text{GAP}(m, s)\). That is \(\sim 7.10\%\).
- For 13 of them \(f(m, s) = \text{TRAIN}(m, s)\). That is \(\sim 0.40\%\).
We are NOT Done

There are problems that none of

FC($m$, $s$), HALF($m$, $s$), INT($m$, $s$), MID($m$, $s$),
EBM($m$, $s$), HBM($m$, $s$), GAP($m$, $s$), TRAIN($m$, $s$)

worked on:

- $f(205, 178)$
- $f(226, 135)$
- $f(233, 141)$
The Scott Huddleston Technique

Scott Huddleston has an algorithm that is REALLY FAST and seems to ALWAYS WORK. Erik and Jacob understand it, nobody else does. They have replicated his results and think that YES it solves ALL problems.

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Lessons Learned

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You never know where the next big project will come from!