1 Quantum Supremacy

The original point of quantum computing is that it should be better than classical computing on some problems. This goal was crystallized by John Preskill [16] (see also [17]) who defined ***Quantum Supremacy***.

Definition 1. *__Quantum Supremacy__ is the goal of finding a problem where a quantum computer can outperform a classical computer. Note that this involves (a) formulating a problem, (b) come up with a quantum algorithm for it, (c) code up that algorithm on a real quantum computer, (d) have an argument (it need not be a proof) that any classical algorithm, when coded up, will do much worse than the quantum algorithm. Note that this is not theoretical. The goal involves real quantum computers versus real classical computers. The goal is that the quantum computer is __better__ than the classical one; however, __better__ may be speed or energy or some other parameter.*

In this chapter we discussed many problems where quantum algorithms seem to do better than any known classical algorithm. However, putting these quantum algorithms on a real quantum computer is difficult. Hence other problems have been suggested as candidates for achieving quantum supremacy. We discuss two problems for which quantum supremacy may have been achieved.

1.1 Quantum Sampling

Problem 1.1. RANDOM CIRCUIT SAMPLING *(*RCS*)*

INSTANCE: A quantum circuit C. C takes n qubits as input and outputs n qubit that are then measured to get an element of $\{0,1\}^n$. Note that if you run C on the same input twice you may not get the same answer. Hence $C(0^n)$ can be viewed as a distribution rather than an element of $\{0,1\}^n$. We call this distribution D. If the quantum circuit is chosen at random then the D is far from uniform.

QUESTION: (Informally) Take enough samples of the circuit to show statistically that D is unlikely to be uniform.

The following are known.

1. Bouland et al. [5] and Aaronson & Chen [4] give evidence that if RCS is classically easy then certain reasonable complexity hypothesis are false. Hence RCS is probably hard classically. We note that this is a theoretical asymptotic result so it may not be as helpful for quantum supremacy as it appears at first glance.

2. On October 23, 2019 Google published a paper [8] which claims to have have solved the 53-bit RCS problem in 200 seconds. They also claim that any classical supercomputer would have taken 10,000 years. The quantum processor used is named ***Sycamore***.

3. IBM [15] (see also Scott Aaronson's blog post [1]) showed how the computation would have taken only 2.5 days on a supercomputer.

4. We will stop here; however, the claims and counter-claims about if Google really did achieve quantum supremacy continue. For intelligent discussions of the issue go to Scott Aaronson's Blog and search for `Quantum Supremacy`.

1.2 Gaussian Boson Sampling

Aaronson & Arkhipov [3] proposed BOSON SAMPLING as a candidate problem for quantum supremacy. We omit the description as it is somewhat technical, They give evidence that if BOSON SAMPLING is easy in one way then the polynomial hierarchy collapses, and if it is easy in another way then (with some assumptions) computing the permanent of a matrix is easy, which also implies that the polynomial hierarchy collapses.

Hamilton et al. [11] introduced a related problem, GAUSSIAN BOSON SAMPLING. They give evidence that if GAUSSIAN BOSON SAMPLING is easy then computing the permanent of a matrix is easy, so the polynomial hierarchy collapses.

The following are known.

1. In December of 2020, a group based in the University of Science and Technology of China (USTC) [14] achieved quantum supremacy by implementing GAUSSIAN BOSON SAMPLING on 76 photons with their photonic quantum computer Jiuzhang. The paper states that to generate the number of samples the quantum computer generates in 2 seconds, a classical supercomputer would require 600 million years of computation. For more information on these results see the articles by Conover [7] and Garisto [10].

2. They later increased the number of photons to 113.

3. Martinez-Cifuentes et al. [12] give reasons why the USTC group may not have actually achieved quantum supremacy.

1.3 Has Quantum Supremacy Happened?

We close with a quote from a blog post of Scott Aaronson [2]. We insert comments in parenthesis for clarity.

> The experiments by Google and USTC and now Xanadu (they worked on a graph problem that has a classical $O(n^3)$ algorithm so the quantum advantage cannot be to large) represent a big step forward for the field, but since they started being done, the classical spoofing attacks have also steadily improved, to the point that whether "quantum computational supremacy" still exists depends on exactly how you define it.
>
> Briefly: If you measure by total operations, energy use, or CO2 footprint, then probably yes, quantum supremacy remains. But if you measure by number of seconds, then it doesn't remain, not if you're willing to shell out for enough cores on AWS (Amazon Web Services) or your favorite supercomputer. And even the quantum supremacy that does remain might eventually fall to, e.g., further improvements due to Gao et al. [9]. For more details, see e.g., the now-published work of Pan, Chen and Zhang [13] or this good popular summary by Adrian Cho [6] for Science.

Scott later argues that what is really needed are new quantum supremacy experiments.

References

[1] S. Aaronson. Quantum supremacy: the glove are off, 2019. https://scottaaronson.blog/?p=4372.

[2] S. Aaronson. Summer 2022 quantum supremacy updates, 2022.
https://scottaaronson.blog/?p=6645.

[3] S. Aaronson and A. Arkhipov. The computational complexity of linear optics. *Theory Comput.*, 9:143–252, 2013.
https://arxiv.org/pdf/1011.3245.pdf.

[4] S. Aaronson and L. Chen. Complexity-theoretic foundations of quantum supremacy experiments. In R. O'Donnell, editor, *32nd Computational Complexity Conference, CCC 2017, July 6-9, 2017, Riga, Latvia*, volume 79 of *LIPIcs*, pages 22:1–22:67. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2017.
https://doi.org/10.4230/LIPIcs.CCC.2017.22.

[5] A. Bouland, B. Fefferman, C. Nirkhe, and U. Vazarani. On the complexity and verifiation of quantum random circuit sampling. *Nature Physics*, 15:159–163, 2018.
https://arxiv.org/abs/1803.04402.

[6] A. Cho. Ordinary computers can beat Google's quantum computers after all. *Science*, 377, 2022.
https://www.science.org/content/article/ordinary-computers-can-beat-google-s-quantum-compu

[7] E. Conover. The new light-based quantum computer Jiuzhang has achieved quantum supremacy. *ScienceNews*, January 16, 2021, 2021.
https://www.sciencenews.org/article/new-light-based-quantum-computer-jiuzhang-supremacy.

[8] F. A. et al. (There are 77 authors). Quantum supremacy ausing a programmable supercomuter processor. *Nature*, 574:505–501, 2019.

[9] X. Gao, M. Kalinowski, C.-N. Chou, M. Lukin, B. Barak, and S. Choi. Limitations of linear cross-entropy as a measure for quantum advantage, 2021.
https://arxiv.org/abs/2112.01657.

[10] D. Garisto. Light-based quantum computer exceeds fastest classical supercomputers. *Scientific American*, December 3, 2020, 2020.
https://www.scientificamerican.com/article/light-based-quantum-computer-exceeds-fastest-cl

[11] C. Hamilton, R. Kruse, L. Sansoni, S. Barkhofen, C. Silverhorn, and I. Jex. Gaussian boson sampling. *Physical Review Letters*, 119, 2017.
https://arxiv.org/pdf/1612.01199.pdf.

[12] J. Martinez-Chfuentes, K. Fonseca-Romero, and N. Quesada. Classical models are a better explanation of the Jiuzhang 1.0 Gaussian boson sampler, 2022.
https://arxiv.org/pdf/2207.10058.pdf.

[13] F. Pan, K. Chen, and P. Zhang. Solving the sampling problem of the Syacmore quantum circuit, 2021.
https://arxiv.org/abs/2111.03011.

[14] J.-W. Pan and . other authors. Quantum computational advantage using photons. *Science*, 370, 2020.
https://arxiv.org/abs/2012.01625.

[15] E. Pednault, J. Gunnels, and G. S. Aaronson. Levarging secondary stoarge to simulate deep 54-qubit Sycamore circuits, 2019.
https://arxiv.org/abs/1910.09534.

[16] J. Preskill. Quantum computing and the entanglement frontier, 2012.
https://arxiv.org/pdf/1203.5813.pdf.

[17] J. Preskill. Why I called it quantum supremacy. *Quanta*, 2019.
https://www.quantamagazine.org/john-preskill-explains-quantum-supremacy-20191002/.