BILL, RECORD LECTURE!!!!

BILL RECORD LECTURE!!!

All 2-Coloring Of the Plane have a Red 2-Stick or Blue 3-stick

Exposition by William Gasarch-U of MD

Credit Where Credit is Due

The main result in these slides is due to Szlam (1999).

Chromatic Number of the Plane: Review

In the prior lecture we proved

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2) ℓ_3 is three colinear points p_1, p_2, p_3 where $d(p_1, p_2) = d(p_2, p_3) = 1$.

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4) Given a $\mathrm{COL}\colon \mathbb{R}^2 \to [2]$ a **Red $\boldsymbol{\ell_k}$** is an ℓ_k where all the points in it are **Red**. Similar for a **Blue $\boldsymbol{\ell_k}$**

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Thm $\forall$ $\mathrm{COL}\colon \mathbb{R}^2 \to [2]$ $\exists$ either a **Red** ℓ_2 or a **Blue** ℓ_2.

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Answer on next slide

For All 2-Colorings of The Plane ...

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Case 1 There exists a **Blue** ℓ_3.

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Let $\mathrm{COL}\colon \mathbb{R}^2 \to [2]$.

Case 1 There exists a **Blue** ℓ_3. Then done.

Case 2 There is no **Blue** ℓ_3.

For All 2-Colorings of The Plane ...

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Case 1 There exists a **Blue** ℓ_3. Then done.

Case 2 There is no **Blue** ℓ_3.

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For All 2-Colorings of The Plane . . .

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We define $\mathrm{COL}'\colon \mathbb{R}^2 \to [3]$ as follows:

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We define $\mathrm{COL}'\colon \mathbb{R}^2 \to [3]$ as follows:

$\mathrm{COL}'(x, y)$ is the least $i \in \{0, 1, 2\}$ such that $\mathrm{COL}(x, y + i) =$ **R**.

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This is well defined because of the case we are in.

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Hence $\mathrm{COL}(x_1, y_1 + i) = $ **R** and $\mathrm{COL}(x_2, y_2 + i) = $ **R**

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Since $d((x_1, y_1), (x_2, y_2)) = 1$, $d((x_1, y_1 + i), (x_2, y_2 + i)) = 1$,

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1) $d((x_1,y_1),(x_2,y_2)) = 1$, and

2) $\mathrm{COL}'(x_1,y_1) = \mathrm{COL}'(x_2,y_2) = i$.

Hence $\mathrm{COL}(x_1,y_1+i) = $ **R** and $\mathrm{COL}(x_2,y_2+i) = $ **R**

Since $d((x_1,y_1),(x_2,y_2)) = 1$, $d((x_1,y_1+i),(x_2,y_2+i)) = 1$,

So (x_1,y_1+1) and (x_2,y_2+1) are a **Red** ℓ_2.

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Since $d((x_1, y_1), (x_2, y_2)) = 1$, $d((x_1, y_1 + i), (x_2, y_2 + i)) = 1$,

So $(x_1, y_1 + 1)$ and $(x_2, y_2 + 1)$ are a **Red** ℓ_2. Done!

Can Prove Result About (ℓ_2, ℓ_4)

Using that the Chromatic Number of the Plane (χ) is ≤ 4 one can easily prove the following:

Thm $\forall\ \mathrm{COL}\colon \mathbb{R}^2 \rightarrow [2]\ \exists$ either a **Red** ℓ_2 or a **Blue** ℓ_4.

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Juhasz prove the above theorem without using $\chi \leq 4$. In fact, he prove the theorem in 1979, 39 years before $\chi \leq 4$ was proven.

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We proved $\mathbb{R}^2 \to (\ell_2, \ell_3)$.

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We proved $\mathbb{R}^2 \rightarrow (\ell_2, \ell_3)$.

We will soon present statements about $\mathbb{R}^2 \rightarrow (\ell_2, \ell_n)$

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Positive Results		

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Szlam, 1999	$(\forall X)[\lvert X\rvert = 3]$ $\mathbb{R}^2 \rightarrow (\ell_2, X)$	$\mathbb{R}^2 \rightarrow (\ell_2, \ell_3)$

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Arman-Tsaturian, 2017	$\mathbb{R}^3 \to (\ell_2, \ell_6)$	NONE

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Conlon-Fox, 2018	$(\forall n \ge 2)$ $\mathbb{R}^n \not\to (\ell_2, \ell_{10^{10}})$	$\mathbb{R}^2 \not\to (\ell_2, \ell_{10^{50}})$

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Open Narrow the gap between $\mathbb{R}^2 \to (\ell_2, \ell_5)$ and $\mathbb{R}^2 \not\to (\ell_2, \ell_{10^{50}})$.

$\mathbb{R}^2 \to (\ell_3, \ell_n)$?

All of the negative results hold for all $n \geq 2$. Our interest is in the $n = 2$ case.

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Currier-Moore-Yip, 2024	$\mathbb{R}^n \not\to (\ell_3, \ell_{20})$
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Currier-Moore-Yip, 2024	$\mathbb{R}^n \not\to (\ell_3, \ell_{20})$
	$\mathbb{R}^n \not\to (\ell_4, \ell_{18})$
	$\mathbb{R}^n \not\to (\ell_5, \ell_{10})$

Open Problems

$\mathbb{R}^2 \to (\ell_3, \ell_n)$?

All of the negative results hold for all $n \geq 2$. Our interest is in the $n = 2$ case.

Author and Year	Result
Positive Results	
Currier-Moore-Yip, 2024	$\mathbb{R}^2 \to (\ell_3, \ell_3)$
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Open Problems

Narrow the gap between $\mathbb{R}^2 \to (\ell_3, \ell_3)$ and $\mathbb{R}^2 \not\to (\ell_3, \ell_{20})$.

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Open Problems

Narrow the gap between $\mathbb{R}^2 \to (\ell_3, \ell_3)$ and $\mathbb{R}^2 \not\to (\ell_3, \ell_{20})$.

The results of Fuhrer-Togh and Currie-Moore-Yip are messy. For some reasonable values of n find nice proofs that $\mathbb{R}^2 \not\to (\ell_3, \ell_n)$.

One More Result

Author and Year	Result
Negative Results	
Erdos et al. See Below	$\mathbb{R}_2 \not\to (\ell_6, \ell_6)$

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Erdös, Graham, Montgomery, Rothchild, Spencer, Strauss.

One More Result

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Erdös, Graham, Montgomery, Rothchild, Spencer, Strauss.

The title of the paper is **Euclidean Ramsey Theorems I (1973)** and was (obviously) an early paper on Euclidean Ramsey Theory.

Open Problems about ℓ_2, ℓ_3

ℓ_2

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ℓ_2

Known $\mathbb{R}^2 \to (\ell_2, \ell_5)$

Open Problems about ℓ_2, ℓ_3

ℓ_2

Known $\mathbb{R}^2 \to (\ell_2, \ell_5)$

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Open Problems about ℓ_2, ℓ_3

ℓ_2
Known $\mathbb{R}^2 \to (\ell_2, \ell_5)$
Known $\mathbb{R}^2 \not\to (\ell_2, \ell_{10^{50}})$
Open Narrow the Gap.

Open Problems about ℓ_2, ℓ_3

$\ell_{\mathbf{2}}$

Known $\mathbb{R}^2 \rightarrow (\ell_2, \ell_5)$

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Open Narrow the Gap.

$\ell_{\mathbf{3}}$

Open Problems about ℓ_2, ℓ_3

ℓ_2
Known $\mathbb{R}^2 \rightarrow (\ell_2, \ell_5)$
Known $\mathbb{R}^2 \not\rightarrow (\ell_2, \ell_{10^{50}})$
Open Narrow the Gap.

ℓ_3
Known $\mathbb{R}^2 \rightarrow (\ell_3, \ell_3)$

Open Problems about ℓ_2, ℓ_3

ℓ_2
Known $\mathbb{R}^2 \rightarrow (\ell_2, \ell_5)$
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Open Narrow the Gap.

ℓ_3
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Open Problems about ℓ_2, ℓ_3

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ℓ_3
Known $\mathbb{R}^2 \rightarrow (\ell_3, \ell_3)$
Known $\mathbb{R}^2 \not\rightarrow (\ell_3, \ell_{20})$
Open Narrow the Gap.

Open Problems about ℓ_4, ℓ_5

ℓ_4

Open Problems about ℓ_4, ℓ_5

ℓ_4

Known (not) Do not know any $a \geq 4$ with $\mathbb{R}^2 \to (\ell_4, \ell_a)$

Open Problems about ℓ_4, ℓ_5

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Open Problems about ℓ_4, ℓ_5

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Might be $\mathbb{R}^2 \not\rightarrow (\ell_4, \ell_4)$ so there is no such a.

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ℓ_6

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No Open Problem.

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Open Get nicer proofs.

Open In some cases get nicer proofs of weaker results.

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Conjectures? None.