Information for Co-Authors and
Table of Contents For Ramsey Lines Project and
LaTeX Samples

Information for Co-Authors

About the Table of Contents

The Table of Contents is on the next three pages after this page. It is not written in stone. We may add or subtract or change things around. But it's a good starting point.

LaTeX macros

Use THIS document's macros for what you write. If you add any macros that's okay but write next to it, commented out, something like *BILL-I added the macro in the line above.* I give examples of what I use that might be unusual later in this document.

Plan

For now, write your own chapters with these macros, and have other people on the team proofread them. (The Measure Chapter is different—I will email about that later.) Once you have a first draft, put it on Overleaf (I will create Overleaf soon). The Overleaf document will also have a file of bib entries so we can use bibtex. We may over time add more bib entries to that file.

Timeline

I am out of town Aug 18-Aug 30 and Sept 6-Sept 13. Once I am back I will work **very hard** on my part–some of which is already written but need to be polished and proofread. I am not teaching in the Fall so that's a good time for me. It would be good if all of you have first drafts soon. Having them before school begins would be great. We will spend the fall proofreading, polishing, and hopefully get some new results (computational ones are likely).

Papers

GARY-YUMO-RYAN have a paper on their measure stuff.
BILL and **NEAL** might submit their expository article on Frankl-Wilson to a journal.
Aside from that, I don't see any other papers coming out of this. I could be wrong.

Added co-author GEN

She is my grad student and will do concrete bounds on $\chi(E^n)$.

Audience

Most of the book is for anyone who knows basic combinatorics, knows how to prove things, and has some math maturity. One chapter will be starred and will require some knowledge of measure theory. Another chapter is starred and will require some knowledge of Ramsey theory.

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This is a table of contents with a name next to who will write that chapter or section indicating who will write it.

Preface BILL

It is the last thing to be written since we need to know what we did before he can write this.

1. **Chapter 1: Introduction BILL**

   This will cover history and context, L_2-L_2, and the notations

   $\mathsf{E}^n \rightarrow (\mathrm{L}_a, \mathrm{L}_b)$ and $\mathsf{E}^n \rightarrow (\mathrm{L}_a, \mathrm{L}_b, \mathrm{L}_c)$.

2. **Chapter 2: The** L_2-L_b **Problem**

   2.1) $\mathsf{E}^2 \rightarrow (\mathrm{L}_2, \mathrm{L}_3)$ **BILL**

   2.2) $\mathsf{E}^2 \rightarrow (\mathrm{L}_2, \mathrm{L}_4)$ **BILL**

   2.3) $\mathsf{E}^2 \rightarrow (\mathrm{L}_2, \mathrm{L}_5)$ using $\chi(E^2) \le 5$ **BILL**

   2.4) $\mathsf{E}^2 \rightarrow (\mathrm{L}_2, \mathrm{L}_5)$ not using $\chi(E^2) \le 5$ **SANDY**. Tsaturian paper.

   2.5) $\mathsf{E}^2 \not\rightarrow (\mathrm{L}_2, \mathrm{L}_{157})$ **SANDY**. Arman-Tsaturian paper.
   By the time we write this, the bound might be improved, possibly by us. We might include some weaker bounds if they are educational.

   2.6) $\mathsf{E}^2 \not\rightarrow (\mathrm{L}_2, \mathrm{L}_{\mathrm{big}})$
   This is the Conlon-Fox paper.

3. **Chapter 3: The** L_3-L_b **Problem**

   3.1) $\mathsf{E}^2 \rightarrow (\mathrm{L}_3, \mathrm{L}_3)$ **CHAEWOON**

   3.2) $\mathsf{E}^2 \not\rightarrow (\mathrm{L}_3, \mathrm{L}_{20})$ **CHAEWOON**
   We may work on improving this. We might include some weaker bounds if they are educational.

   3.3) $\mathsf{E}^2 \not\rightarrow (\mathrm{L}_3, \mathrm{L}_{\mathrm{big}})$ **BILL**
   This is the Conlon-Wu paper.

4. **Chapter 4: The** L_4-L_b **Problem CHAEWOON** and **GARY**

   4.1) $\mathsf{E}^2 \not\rightarrow (\mathrm{L}_4, \mathrm{L}_{14})$
   GARY claims that the original result, which is $(\mathrm{L}_4, \mathrm{L}_{18})$, can be improved to $(\mathrm{L}_4, \mathrm{L}_{14})$. We may work on improving this further.

5. **Chapter 5: The** L_5-L_b **Problem CHAEWOON** and **GARY**

   5.1) $\mathsf{E}^2 \not\rightarrow (\mathrm{L}_5, \mathrm{L}_8)$
   GARY claims that the original result, which is $(\mathrm{L}_5, \mathrm{L}_{10})$, can be improved to $(\mathrm{L}_5, \mathrm{L}_8)$. We may work on improving this.

6. **Chapter 6: The** L_6-L_b **Problem KELIN**

   6.1) $\mathsf{E}^2 \not\rightarrow (\mathrm{L}_6, \mathrm{L}_6)$

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7. **Chapter 7: Measurable Colorings* GARY-YUMO-RYAN**

$\mathsf{E}^n \rightarrow_m (\mathrm{L}_a, \mathrm{L}_b)$ means that, for every *measurable 2-colorings*, there is either a red L_a or a blue L_b.

Let Sq_d be a square with side length d. A mono Sq_d has all vertices the same color.

Let Pent_d be a pentagon with side length d. A mono Pent_d has all vertices the same color.

Let Hex_d be a hexagon with side length d. A mono Hex_d has all vertices the same color.

Let Hep_d be a heptagon with side length d. A mono Hep_d has all vertices the same color.

I do not yet know how to divide this chapter into sections, so I just list the results.

(a) $\mathsf{E}^2 \rightarrow_m (\mathrm{L}_2, \mathrm{L}_7)$

(b) $\mathsf{E}^2 \rightarrow_m (\mathrm{L}_2, \mathrm{L}_2, \mathrm{L}_3)$

(c) $\mathsf{E}^3 \rightarrow_m (\mathrm{L}_3, \mathrm{L}_4)$

(d) $\mathsf{E}^4 \rightarrow_m (\mathrm{L}_3, \mathrm{L}_5)$

(e) $\mathsf{E}^7 \rightarrow_m (\mathrm{L}_3, \mathrm{L}_6)$

(f) $\mathsf{E}^4 \rightarrow_m (\mathrm{L}_4, \mathrm{L}_4)$

(g) $\mathsf{E}^2 \rightarrow_m (\mathrm{L}_2, \mathrm{Sq}_{\sqrt{2}})$
The center of the square also has that color. This result is in contrast to Csizmadia-Toth's result that $\mathsf{E}_2 \not\rightarrow (\mathrm{L}_2, \mathrm{Hep}_{9/5\sin(\pi/7)})$ and the center is also that color.
BILL TO GARY - I DO NOT SEE HOW THAT IS A CONTRAST.

(h) $\mathsf{E}^2 \rightarrow_m (\mathrm{L}_2, \mathrm{Pent}_1)$

(i) $\mathsf{E}^2 \rightarrow_m (\mathrm{L}_2, \mathrm{Hex}_1)$

(j) For all 2-colorings of E^3 there is either a mono Sq_1 or a mono (irregular) tetrahedron with edge lengths $(1, 1, 1, 1, \sqrt{2}, \sqrt{2})$.

GARY, **YUMO**, **RYAN**- we might leave out the results having to do with squares, pentagons, etc.

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8. **Chapter 8: The Chromatic Number of E^n: Asymptotics**
   BILL and **NEAL**

   8.1) $\chi(\mathsf{E}^n) \le$ easy upper bound

   8.2) $\chi(\mathsf{E}^n) \ge$ easy lower bound

   8.3) $\chi(\mathsf{E}^n) = \Omega(n^2)$

   8.4) For all d, $\chi(\mathsf{E}^n) = \Omega(n^d)$

   8.5) $\chi(\mathsf{E}^n) \ge (1 + o(1))(1.2)^n$

   8.6) Whatever the best upper and lower bounds are. We will decide later if we just want to state them or if we want to prove them.

9. **Chapter 9: The Chromatic Number of E^n: Concrete Results GEN**

10. **Chapter 10: Application to Ramsey Theory* BILL** and **NEAL**

    10.1) $R(k) \le 2^{2k}/\sqrt{k}$

    10.2) Probabilistic proof that $R(k) = \Omega(k2^{k/2})$

    10.3) Constructive proof that $R(k) = \Omega(k^3)$

    10.4) Constructive proof that, for all d, $R(k) = \Omega(k^d)$

    10.5) Constructive proof that $R(k) = \exp((1 + o(1))\frac{\log^2 k}{4 \log\log k})$

    10.6) Survey of what else is known about constructive proofs of lower bounds on $R(k)$.

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LaTeX Samples

You need to be looking at the LaTeX for this document to follow what I am saying.

1 Theorems and Lemmas and Proofs, Oh My!

To write a theorem and a proof, do the following.

Theorem 1.1 *REU-CAAR 2026 was awesome.*

Proof:

Assume that this is not an awesome theorem. Then every 6-7 cardinal is actually an I0-I1 cardinal. This is obviously false. Hence this theorem is awesome.

∎

If you look at the LaTeX code, you will see that I labeled the theorem **th:awesome**. This is my convention, which you should follow: Theorems are labeled with a string that begins th:

Here is how you refer to a labeled theorem:

Theorem 1.1 states that REU-CAAR 2026 was awesome.

I now give you lemma **(le:)**, corollary **(co:)**, definition **(de:)**, notation **(no:)**, note **(note:)**, and **conjecture (conj:)**.

Lemma 1.2 *This is an awesome lemma.*

Definition 1.3 This is an awesome definition.

Notation 1.4 This is an awesome notation.

Note 1.5 This is an awesome note.

Conjecture 1.6 *This is an awesome conjecture.*

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2 Colorings, What to Color, and Names of Colors

In this section, when I am refering to a macros, I put it in boldface even though the macro itself does not produce boldface. For example: The macro for N is **N**.

1. If we are 2-coloring, use the domain [2], but still refer to the colors as RED and BLUE.
2. If we are c-coloring, use the domain $[c]$.
3. We will always be coloring E^n, usually E^2.
4. The macros for the natural numbers is **N**.
5. The macro for rationals is **Q**.
6. The macros forthe real numbers is **reals**, not **R**, since **R** is for RED.
7. The macros for the complex numbers is **complex**, not **C**, since **C** stands fr something else in LaTeX. I doubt we will use **complex** anyway.
8. The macro for L is **LL**. This gives a roman font L. It is always used in math mode. We could not use **L** since that's a macro for something else in LaTeX.
9. I have a macro for $\mathrm{L}_{\mathrm{big}}$ which is **Lbig**.

3 Colors and Coloring

1. We use COL for coloring. Hence we commonly say

   Let $\mathrm{COL}\colon \mathsf{E}^2 \to [2]$.
2. We have defined macros for RED, BLUE, and GREEN as **R**, **B**, **G**. Always use these macros in math mode, since that gives good spacing, and they are often used in math mode anyway.

   For example, we might say:

   Let $\mathrm{COL}\colon \mathsf{E}^2 \to [2]$. Let T be an equilateral triangle. Two of the vertices must have the same color, say RED. These two form a RED L_2.

4 People's Names With Accents

I have macros for several people's names that contain accents. For example
1) **Erdos** is the macro for Erdős with a space after the last letter. So this is fine for

Erdős and Gasarch proved the Goldbach conjecture.

2) **Erdosns** is the macro for Erdős with no space after the last letter.
Here is an example of a sentence that uses both macros.

Erdős made many conjectures. Some were with other people such as the Erdős–Turán conjecture.

Note that for the "Erdős–Turán conjecture" you want to have NO space after the last letter in Erdős. OH, same in that last sentence since after the name was a period.