Duels

William Gasarch-U of MD

Set Up

Set Up

1. Alice has prob p_a of killing Bob.

Set Up

1. Alice has prob p_a of killing Bob.
2. Alice has n_a bullets. $n_a = \infty$ is possible.

Set Up

1. Alice has prob p_a of killing Bob.
2. Alice has n_a bullets. $n_a = \infty$ is possible.
3. Alice goes first.

Set Up

1. Alice has prob p_a of killing Bob.
2. Alice has n_a bullets. $n_a = \infty$ is possible.
3. Alice goes first.
4. Bob has prob p_b of killing Alice.

Set Up

1. Alice has prob p_a of killing Bob.
2. Alice has n_a bullets. $n_a = \infty$ is possible.
3. Alice goes first.
4. Bob has prob p_b of killing Alice.
5. Bob has n_b bullets. $n_b = \infty$ is possible.

Set Up

1. Alice has prob p_a of killing Bob.
2. Alice has n_a bullets. $n_a = \infty$ is possible.
3. Alice goes first.
4. Bob has prob p_b of killing Alice.
5. Bob has n_b bullets. $n_b = \infty$ is possible.

What is the prob that Alice kills Bob?

The Case of Infinite Bullets

If $n_a = n_b = \infty$ then what is the prob that Alice kills Bob.

The Case of Infinite Bullets

If $n_a = n_b = \infty$ then what is the prob that Alice kills Bob.

Prob Alice kills Bob in her first shoot is p_a.

The Case of Infinite Bullets

If $n_a = n_b = \infty$ then what is the prob that Alice kills Bob.

Prob Alice kills Bob in her first shoot is p_a.

Prob Alice kills Bob in her second shot is prob Alice misses, then Bob misses, then Alice hits. This is

The Case of Infinite Bullets

If $n_a = n_b = \infty$ then what is the prob that Alice kills Bob.

Prob Alice kills Bob in her first shoot is p_a.

Prob Alice kills Bob in her second shot is prob Alice misses, then Bob misses, then Alice hits. This is

$(1 - p_a)(1 - p_b)p_a$.

The Case of Infinite Bullets

If $n_a = n_b = \infty$ then what is the prob that Alice kills Bob.

Prob Alice kills Bob in her first shoot is p_a.

Prob Alice kills Bob in her second shot is prob Alice misses, then Bob misses, then Alice hits. This is

$(1 - p_a)(1 - p_b)p_a$.

Prob Alice kills Bob in her Third shot is prob Alice misses, then Bob misses, then Alice misses, then Bob misses, then Alice hits.

The Case of Infinite Bullets

If $n_a = n_b = \infty$ then what is the prob that Alice kills Bob.

Prob Alice kills Bob in her first shoot is p_a.

Prob Alice kills Bob in her second shot is prob Alice misses, then Bob misses, then Alice hits. This is

$(1 - p_a)(1 - p_b)p_a$.

Prob Alice kills Bob in her Third shot is prob Alice misses, then Bob misses, then Alice misses, then Bob misses, then Alice hits.

$(1 - p_a)(1 - p_b)(1 - p_a)(1 - p_b)p_a = ((1 - p_a)(1 - p_b))^2 p_a$.

The Case of Infinite Bullets

If $n_a = n_b = \infty$ then what is the prob that Alice kills Bob.

Prob Alice kills Bob in her first shoot is p_a.

Prob Alice kills Bob in her second shot is prob Alice misses, then Bob misses, then Alice hits. This is

$(1 - p_a)(1 - p_b)p_a$.

Prob Alice kills Bob in her Third shot is prob Alice misses, then Bob misses, then Alice misses, then Bob misses, then Alice hits.

$(1 - p_a)(1 - p_b)(1 - p_a)(1 - p_b)p_a = ((1 - p_a)(1 - p_b))^2 p_a$.

Prob Alice kills Bob in her ith shot

The Case of Infinite Bullets

If $n_a = n_b = \infty$ then what is the prob that Alice kills Bob.

Prob Alice kills Bob in her first shoot is p_a.

Prob Alice kills Bob in her second shot is prob Alice misses, then Bob misses, then Alice hits. This is

$(1 - p_a)(1 - p_b)p_a$.

Prob Alice kills Bob in her Third shot is prob Alice misses, then Bob misses, then Alice misses, then Bob misses, then Alice hits.

$(1 - p_a)(1 - p_b)(1 - p_a)(1 - p_b)p_a = ((1 - p_a)(1 - p_b))^2 p_a$.

Prob Alice kills Bob in her ith shot

$((1 - p_a)(1 - p_b))^{i-1} p_a$.

The Case of Infinite Bullets

If $n_a = n_b = \infty$ then what is the prob that Alice kills Bob.

Prob Alice kills Bob in her first shoot is p_a.

Prob Alice kills Bob in her second shot is prob Alice misses, then Bob misses, then Alice hits. This is

$(1 - p_a)(1 - p_b)p_a$.

Prob Alice kills Bob in her Third shot is prob Alice misses, then Bob misses, then Alice misses, then Bob misses, then Alice hits.

$(1 - p_a)(1 - p_b)(1 - p_a)(1 - p_b)p_a = ((1 - p_a)(1 - p_b))^2 p_a$.

Prob Alice kills Bob in her ith shot

$((1 - p_a)(1 - p_b))^{i-1} p_a$.
Prob Alice kills Bob is

$$\sum_{i=1}^{\infty} ((1 - p_a)(1 - p_b))^{i-1} p_a = p_a \sum_{i=1}^{\infty} ((1 - p_a)(1 - p_b))^{i-1}$$

The Case of Infinite Bullets

If $n_a = n_b = \infty$ then what is the prob that Alice kills Bob.

Prob Alice kills Bob in her first shoot is p_a.

Prob Alice kills Bob in her second shot is prob Alice misses, then Bob misses, then Alice hits. This is

$(1 - p_a)(1 - p_b)p_a$.

Prob Alice kills Bob in her Third shot is prob Alice misses, then Bob misses, then Alice misses, then Bob misses, then Alice hits.

$(1 - p_a)(1 - p_b)(1 - p_a)(1 - p_b)p_a = ((1 - p_a)(1 - p_b))^2 p_a$.

Prob Alice kills Bob in her ith shot

$((1 - p_a)(1 - p_b))^{i-1} p_a$.

Prob Alice kills Bob is

$$\sum_{i=1}^{\infty} ((1 - p_a)(1 - p_b))^{i-1} p_a = p_a \sum_{i=1}^{\infty} ((1 - p_a)(1 - p_b))^{i-1}$$

Need to sum that infinite series.

Infinite Series

Infinite Series

$$S = \sum_{i=1}^{\infty} x^{i-1}$$

Infinite Series

$$S = \sum_{i=1}^{\infty} x^{i-1}$$

Better to sum from $i = 0$. Let $j = i - 1$

Infinite Series

$$S = \sum_{i=1}^{\infty} x^{i-1}$$

Better to sum from $i = 0$. Let $j = i - 1$

$S = \sum_{j=0}^{\infty} x^j$

Infinite Series

$$S = \sum_{i=1}^{\infty} x^{i-1}$$

Better to sum from $i = 0$. Let $j = i - 1$

$S = \sum_{j=0}^{\infty} x^j$

$xS = \sum_{j=1}^{\infty} x^j$

Infinite Series

$$S = \sum_{i=1}^{\infty} x^{i-1}$$

Better to sum from $i = 0$. Let $j = i - 1$

$S = \sum_{j=0}^{\infty} x^j$

$xS = \sum_{j=1}^{\infty} x^j$

Subtract to get

$S(1 - x) = 1$

$S = \frac{1}{1-x}$.

Recap and Finish Up Infinity Case

Recap last slide:
$\sum_{i=1}^{\infty} x^{i-1} = \frac{1}{1-x}$

Recap and Finish Up Infinity Case

Recap last slide:
$\sum_{i=1}^{\infty} x^{i-1} = \frac{1}{1-x}$
Hence

Recap and Finish Up Infinity Case

Recap last slide:
$\sum_{i=1}^{\infty} x^{i-1} = \frac{1}{1-x}$
Hence

$$p_a \sum_{i=1}^{\infty} ((1-p_a)(1-p_b))^{i-1} = \frac{p_a}{1-(1-p_a)(1-p_b)}$$

First Research Question

First Research Question

First Research Question Going first is an advantage. How much better a shot does Bob have to be to negate that advantage.

First Research Question

First Research Question Going first is an advantage. How much better a shot does Bob have to be to negate that advantage.

Make a table of the following:
As $p_a = 0.1, 0.2, \ldots, 0.9$ see what value of p_b makes

$$\frac{p_a}{1-(1-p_a)(1-p_b)} = 0.5.$$

First Research Question

First Research Question Going first is an advantage. How much better a shot does Bob have to be to negate that advantage.

Make a table of the following:
As $p_a = 0.1, 0.2, \ldots, 0.9$ see what value of p_b makes

$$\frac{p_a}{1-(1-p_a)(1-p_b)} = 0.5.$$

This will give a sense of how important going first is.

Second Research Question

Second Research Question

Second Research Question What if they shoot at the same time so there is no **First Player**? Then what is the prob of that Alice kills Bob?

Recurrence

Warning- this is not quite correct since it is possible that nobody dies.

Recurrence

Warning- this is not quite correct since it is possible that nobody dies.
Alice has prob p_A of killing Bob and x_A bullets.

Recurrence

Warning- this is not quite correct since it is possible that nobody dies.
Alice has prob p_A of killing Bob and x_A bullets.
Bob has prob $p_B$of killing Alice and x_B bullets.
Alice goes first.

Recurrence

Warning- this is not quite correct since it is possible that nobody dies.
Alice has prob p_A of killing Bob and x_A bullets.
Bob has prob $p_B$of killing Alice and x_B bullets.
Alice goes first.
$\mathrm{Pr}_A(p_A, p_B, x_A, x_B)$ is prob that Alice kills Bob.

Recurrence

Warning- this is not quite correct since it is possible that nobody dies.
Alice has prob p_A of killing Bob and x_A bullets.
Bob has prob $p_B$of killing Alice and x_B bullets.
Alice goes first.
$\Pr_A(p_A, p_B, x_A, x_B)$ is prob that Alice kills Bob.
$\Pr_B(p_A, p_B, x_A, x_B)$ is prob that Bob kills Alice.
It is the sum of the following:

Recurrence

Warning- this is not quite correct since it is possible that nobody dies.
Alice has prob p_A of killing Bob and x_A bullets.
Bob has prob $p_B$of killing Alice and x_B bullets.
Alice goes first.
$\mathrm{Pr}_A(p_A, p_B, x_A, x_B)$ is prob that Alice kills Bob.
$\mathrm{Pr}_B(p_A, p_B, x_A, x_B)$ is prob that Bob kills Alice.
It is the sum of the following:

- Prob that Alice shoots and wins. Thats p_A.

Recurrence

Warning- this is not quite correct since it is possible that nobody dies.
Alice has prob p_A of killing Bob and x_A bullets.
Bob has prob $p_B$of killing Alice and x_B bullets.
Alice goes first.
$\mathrm{Pr}_A(p_A, p_B, x_A, x_B)$ is prob that Alice kills Bob.
$\mathrm{Pr}_B(p_A, p_B, x_A, x_B)$ is prob that Bob kills Alice.
It is the sum of the following:

- Prob that Alice shoots and wins. Thats p_A.
- Prob that Alice misses, the Bob misses, then both have one less bullet and Alice wins. Thats

Recurrence

Warning- this is not quite correct since it is possible that nobody dies.
Alice has prob p_A of killing Bob and x_A bullets.
Bob has prob $p_B$of killing Alice and x_B bullets.
Alice goes first.
$\mathrm{Pr}_A(p_A, p_B, x_A, x_B)$ is prob that Alice kills Bob.
$\mathrm{Pr}_B(p_A, p_B, x_A, x_B)$ is prob that Bob kills Alice.
It is the sum of the following:

- Prob that Alice shoots and wins. Thats p_A.
- Prob that Alice misses, the Bob misses, then both have one less bullet and Alice wins. Thats

$$(1 - p_A)(1 - p_B)\mathrm{Pr}_A(p_A, p_B, x_A - 1, x_B - 1).$$

We need something else to make this work: What if Alice or Bob has no bullets?

Recurrence: Need Base Case