

High School Projects In Computer Science

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NIM Games

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Play with 10 stones on white board.

NIM Games-Win Table for 1,2,3

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n	0	1	2	3	4	5	6	7	8	9	10	11	12
W													

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W	//	/	/	/	//	/	/	/	//	/	/	/	//

Player II wins iff $n \equiv 0 \pmod{4}$.

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Work on the win table for 1,3,4 together. I give you 5 minutes

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W															

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W	//	/	//												

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Player I wins iff $n \equiv 0, 2 \pmod{7}$.

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Player II wins $1, x, x + 1$ IFF $n \equiv \text{BLAH} \pmod{\text{BLAHBLAH}}$.
Might involve cases like x even or odd.

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4. Same for $1, x, x + 2$ and others.
5. (Optional) Automate the process

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6. Very good for HS projects where the specs wants lots of stats like a science experiment.

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3. **Project** Answer the following question: If your AI is going to play against p -players then what is the q such that you should train it against q -players.

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4. Nim With Cash-Opponent Version: **Example** NIM(1,2,3). Player 1 begins with x dollars, Player 2 with y dollars. Each player can remove 1 or 2 or 3 stones. If a player removes x stones he gives x dollars to his opponent.

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Two players RED and BLUE.

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- ▶ They keep doing this until there is either a RED or BLUE triangle.

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Unlike NIM, there are NOT known strategies for Ramsey Games (though the six point version is probably known.)

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The Ramsey Game projects are really math.

Duels and Bullets

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Given (p_A, p_B, a, b) who has the advantage.

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Code up, collect data, make conjectures.

Primes in Other Domains

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Look at this and other domains to investigate

How many primes are $\leq n$? How does that compare to the normal numbers?

Do we have Unique Factorization?

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which one grows faster? What do they look like? What matters more the subscript or the additive term?

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Known If look at the equation mod 2,3,5,7 then it hits 0 infinitely often.

Open What about other mods?

Project Gather evidence for conjectures. Also vary initial conditions.

SAT Solvers

Given a boolean formula like

$$(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_4)$$

We want to know if it is SATISFIABLE. There are many algorithms for this. Code them up, see how they do.

Crypto

William Gasarch- U of MD

Crypto Projects

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- (4) Coding up discrete log and Diffie-Helman.

Complexity Theory

(Needs background so you might not understand this slide.)

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- (1) Code up SAT Solvers. Apply VV to it.
- (2) DFA-tricks for division VS just doing the division.
- (3) Do more queries help: 3-SUM, APSP.

Coloring and Functions

Let f be a function from N to N (we take N to NOT have 0). We use f to color the integers.

- ▶ Color n **RED** if it is the value of f on a **BLUE** number
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Example $f(x) = x + 1$.

1 is NOT in the f of ANYTHING so its NOT f on a **BLUE** .

Hence $COL(1) = BLUE$.

2 IS $f(1)$ and 1 is **BLUE** , so 2 is **RED**

3 is $f(2)$ but 2 is **RED** , so 3 is **BLUE**

...

So we get an alternating pattern.

What happens with other functions? With more complicated rules?