Exceprty from The Book Review Column[1]

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In this column we review the following books.

1. **Proceedings from the Gatherings for Gardner Conference**. Edited by a variety of people. To show their appreciation for Martin Gardner there have been several *Gatherings for Gardner* conferences which are gatherings of mathematicians, puzzlers, and magicians. The first two gatherings were in 1993 and 1995. They had no proceedings. However, the ones since 1998 have. We give reviews of the six proceedings that are available. Review by Bill Gasarch.

2. **Comprehensive Mathematics for Computer Scientists 1** by Guerino B. Mazzola, Gerard Milmeister, Jody Weissmann. The reviewed volume (part 1 of 2) covers the basic theory of sets and numbers, algebra, graphs, automata, and linear geometry. The coverage is intentionally broad rather than deep, aimed at college level students. Review by Sage LaTorra.

3. **Creative Mathematics** by H.S. Wall. The *Moore Method* is a method of teaching where you let the students discover theorems for themselves rather than guide them. This may be an exaggeration; however, you try to have them do this. Does it work? This is a textbook for teaching calculus this way. Review by Jason Dyer.

4. **Nonlinear Integer Programming** by Duan Li and Xiaoling Sun. How do you optimize a nonlinear function with nonlinear constraints? This problem is, in general, NP-hard. But people still need to really do this. How do they? Read this book and find out. Review by Justin Melvin.

5. **Complex Social Networks** by Fernando Vega-Redondo. To quote the review itself: This is a review, written by a computational biologist, for a CS theory newsletter, about a book in an economics monograph series on the subject of complex social networks. This confluence of disciplines illustrates how the subject of the reviewed book has become central to many fields. Reviewed by Carl Kingsford.

6. **The Calculus of Computation:Decision Procedures with Applications to Verification** by Aaron R. Bradley and Zohar Manna. The book covers three main areas, tightly interconnected: mathematical logic, proofs of correctness for sequential programs, and decision procedures for a few decidable logic theories. Review by Carlo A. Furia.

7. **Algorithms on Strings** by Crochemore, Hanchart and Lecroq. The title says it all. Review by Yiorgos Adamopoulos.

8. **A View from the Top:Analysis, Combinatorics and Number Theory** by Alex Iosevich. This is an undergraduate-level math textbook. It is quite special because instead of specializing on a given topic, it wanders through fields as different as Calculus, Number

Theory, Probability, Algebra and Geometry. Does the approach work? Read the review and find out. Or just read the book. Review by Yannis Haralambous.

9. **Geometric Folding Algorithms** by E.D. Demaine and J. O'Rourke. *Geometric Folding Algorithms* presents a comprehensive discussion of folding (and unfolding) algorithms in Euclidean space. The book is divided into three parts: Linkages, Paper, and Polyhedra. The target audience for this book is a graduate or undergraduate student in computer science or mathematics. In addition, the book is so visually appealing that it can be used as a coffee table conversation piece. But do not be fooled- there is serious math of interest here as well. Review by Brittany Terese Fasy and David L. Millman.

10. **Geometric Algebra: An Algebraic System for Computer Games and Animation** by Author: J. Vince. Geometric algebra is a framework where geometric objects (lines, areas, volumes, etc.) are treated as vectors. In doing so, we are able to generalize familiar ideas of scalar and cross products of vectors to the inner and outer products of geometric objects. The central concept behind GA is the geometric product, the sum of the inner and outer products. Review by Brittany Terese Fasy and David L. Millman.

11. **Dude, Can You Count?** by Christian Constanda. This book is a vigorous criticism of many things in our society. Some of them are mathematical, some of them are not; however, mathematics permeates the entire book. Review by Bill Gasarch.

BOOKS I NEED REVIEWED FOR SIGACT NEWS COLUMN
Algorithms and Related Topics

1. *Proofs and Algorithms* by Gilles Dowek.
2. *Flows in Networks* by Ford and Fulkerson. (the classic reprinted!)
3. *Algorithmic Algebraic Combinatorial and Grobner Bases* Edited by Klin, Jones, Jurisic, Muzychuk, Ponomarnko.
4. *Decision Procedures* by Kroening and Strichman.
5. *Computational Topology* by Edelsbrunner and Harer.
6. *Fast Algorithms for Signal Processing* by Blahut.
7. *Information Retrieval: Implementing and Evaluating Search Engines* by Buttcher, Clarke, and Cormack.
8. *Bioinspired Computation in Combinatorial Optimization* by Neumann and Witt.
9. *Exact Exponential Algorithms* by Fomin and Kratsch.
10. *Modern Computer Arithmetic* by Brent and Zimmermann.

Cryptography, Coding Theory, Security

1. *The Cryptoclub: Using Mathematics to Make and Break Secret Codes* by Beissinger and Pless (for middle school students).

2. *Mathematical Ciphers: From Ceaser to RSA* by Anne Young. (For a non-majors course.)

3. *Adaptive Cryptographic Access Control* by Kayem, Akl, and Martin.

4. *Preserving Privacy in data outsourcing* by Sara Foresti.

5. *Encryption for Digital Content* by Kiayias and Pehlivanoglu.

6. *Cryptanalysis of RSA and its variants* by Hinek.

7. *Algorithmic Cryptanalysis* by Joux.

Combinatorics, Probability, and Number Theory

1. *Elementary Probability for Applications* by Durrett

2. *Combinatorial Pattern Matching Algorithms in Computational Biology Using Perl and R* by Valiente.

3. *Biscuits of Number Theory* Edited by Author Benjamin and Ezra Brown.

Complexity Theory

1. *Elements of Computation Theory* by Singh (ugrad text)

2. *Deterministic Extraction from Weak Random Sources* by Ariel Gabizon.

3. *Grammatical Inference: Learning Automata and Grammars* by Colin de la Higuera.

Misc-Serious Math

1. *Polynomia and Related Realms* by Dan Kalman.

2. *Mathematica: A problem centered approach* by Hazrat.

3. *Finite Fields and Applications* by Mullen and Mummert.

4. *Mathematics Everywhere* Edited by Aigner and Behrends.

5. *Mathematical Omnibus: Thirty Lectures on Classic Mathematics* by Fuchs and Tabachnikov.

6. *Roots to Research: A Vertical Development of Mathematical Problems* by Sally and Sally.

7. *Probability Theory: An Analytic View* by Stroock.

Misc-Fun Math

1. *The Dots and Boxes Game: Sophisticated Child's play* By Berlekamp.

2. *New Mathematical Diversions* by Martin Gardner.

Misc-Comp Sci

1. *Foundations of XML Processing: The Tree-Automata Approach* by Haruo Hosoya.
2. *Introduction to Computational Proteomics* by Yona.
3. *Network Security: A Decision and Game-Theoretic Approach* by Alpcan and Basar.
4. *Introduction to the Theory of Programming Languages* by Dowek and Levy.
5. *Logical Analysis of Hybrid Systems: Proving Theorems for Complex Dynamics* by Platzer.
6. *Digital Nets and Sequences: Discrepancy theory and Quasi-Monte Carlo Integration* by Josef Dick and Friedrich Pillichshammer.
7. *Approximation and Computation: In Honors of Gradimir Milovanovic* Edited by Gautchi, Mastoianni, Rassias.
8. *Clustering in Bioinformatics and Drug Discovery* by MacCuish and MacCuish.
9. *Drawing Programs: The theory and Practice of Schematic Functional Programming* by Addis and Addis.

History of Math

1. *History of Mathematics: Highways and Byways* by Amy Dahan-Dalmedico and Jeanne Peilfer.
2. *An Episodic History of Mathematics: Mathematical Culture through Problem Solving* by Steven Krantz.

Joint Review of[2] of
The Mathemagician and Pied Piper:
A Collection in Tribute to Martin Gardner
Edited by Elywn Berlekamp and Tom Rodgers
Published by A.K.Peters, 1999
265 pages, Hardcover, $40.00

Puzzlers' Tribute:
A Feast for the Mind
Edited by David Wolfe and Tom Rodgers
Published by A.K.Peters, 2002
420 pages, Hardcover, $40.88

Tribute to a Mathemagician
Edited by Barry Cipra, Erik Demaine, Martin Demaine, Tom Rodgers
Published by A.K.Peters, 2004
262 pages, Hardcover, $40.00

Mathematical Wizardry for a Gardner
Edited by Ed. Pegg Jr, Alan Schoen, Tom Rodgers
Published by A.K.Peters, 2005
262 pages, Hardcover, $40.00

A Lifetime of Puzzles:
A Collection of Puzzles in Honor of Martin Gardner's 90th Birthday
Edited by Erik Demaine, Martin Demaine, Tom Rodgers
Published by A.K.Peters, 2008
262 pages, Hardcover, $40.00

Homage to a Pied Puzzler
Edited by Ed. Pegg Jr, Alan Schoen, Tom Rodgers
Published by A.K.Peters, 2009
262 pages, Hardcover, $50.00
Review by
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1 Introduction

Martin Gardner is well known for writing a Math Column in Scientific American from 1956 to 1981. His influence on mathematics is immense. Many (including me) have realized that mathematics can be fun and interesting via his columns. Alas he passed away at the age of 95 in 2010.

To show their appreciation there have been several *Gatherings for Gardner* which are gatherings of mathematicians, puzzlers, and magicians. They give talks on work that is related to what Gardner has written about- recreational mathematics which sometimes connects to more serious topics. The first two gatherings were in 1993 and 1995. They had no proceedings. However, the ones since 1998 have. We give reviews of the six proceedings that are available.

All of the proceedings have a similar character. They all have a large variety of types of articles: math problems, recreational math, history of math or puzzles or magic, and some personal stories about Martin Gardner and other mathematicians. One proceeding had an article by Frank Harary, and a later one had an article about him in Memoriam

The writing quality of the articles is surprisingly good for a collection of articles by different people. Some topics intrigued me and others did not; however, when a topic did not intrigue me it may well intrigue someone else.

I will review each book separately; however, since they are similar (this is not a criticism) I will give (roughly) less detail on a book the further down the list they are. I will then review them as a whole.

2 The Mathemagician and Pied Puzzler

Personal Magic: This section has chapters about Martin Gardner directly.

Martin Gardner: "A Documentary" by Dana Richards is a documentary in words. It is a collection of facts and quotes about and by Martin Gardner from various interviews. In just 10 pages this tells how he got to be where he is and what kind of man he was. *Ambrose, Gardner,and Doyle* by Raymond Smullyan is a science fiction story about people in the future looking back at an article that Martin Gardner had written. I would call it a satire of academia except that it is too close to being true.

Puzzlers

Card Game Trivia by Stewart Lamle is a one page list of facts about cards. Here is one: The Joker was devised by a Mississippi riverboat gambler to increase the odds of getting good poker hands. *Creative Puzzle Thinking* by Nob Yoshigahara is a list of 11 puzzles without the answers. Here is an example:

Which two numbers come at the end of this sequence?

2,4,5,30,32,34,36,40,42,44,46,50,52,54,56,60,62,64,x,y

Number Play, Calculators, and Card Tricks: Mathemagical Black Holes by Michael W. Ecker. Take a number, say 100. Now write down all of its divisors (include 1 and the number itself): $1, 2, 4, 5, 10, 20, 25, 50, 100$. Now take the sum of the digits of the divisors: $1 + 2 + 4 + 5 + 1 + 0 + 2 + 0 + 2 + 5 + 5 + 0 + 1 + 0 + 0 = 28$.

Now repeat the process. The divisors of 28 are $1, 2, 4, 7, 14, 28$. The sum of the digits of the divisors is $1 + 2 + 4 + 7 + 1 + 4 + 2 + 8 = 29$.

The divisors of 29 are $1, 29$. The sum of the digits of the divisors is $1 + 2 + 9 = 12$.

The divisors of 12 are $1, 2, 3, 4, 6, 12$. The sum of the digits of the divisors is $1+2+3+4+6+1+2 = 19$.

The divisors of 19 are $1, 19$. The sum of the digits of the divisors is $1 + 1 + 9 = 11$.

The divisors of 11 are $1, 11$. The sum of the digits of the divisors is $1 + 1 + 1 = 3$.

The divisors of 3 are $1, 3$. The sum of the digits of the divisors is $1 + 3 = 4$.

The divisors of 4 are $1, 2, 4$. The sum of the digits of the divisors is $1 + 2 + 4 = 7$.

The divisors of 7 are $1, 7$. The sum of the digits of the divisors is $1 + 7 = 8$.

The divisors of 8 are $1, 2, 4, 8$. The sum of the digits of the divisors is $1 + 2 + 4 + 8 = 15$.

The divisors of 15 are $1, 3, 5, 15$. The sum of the digits of the divisors is $1 + 2 + 4 + 8 = 15$.

AH-Ha- 15 goes to itself. Of more interest is that *every number eventually goes to 15.* This is called a *black hole.* This chapter discusses many such black holes and applies them to a magic trick.

Puzzles From Around the World by Richard Hess. This chapter has 17 easy problems, 19 medium problems, 20 hard problems, and solutions to all of them. Here is a Hard Problem:
An irrational punch centered on point p in the plane removes all points that are an irrational distance from p. How many irrational punches are needed to remove all points from the plane.

Mathemagics

Misfiring Tasks by Ken Knowlton. There are theorems in math that hold for all but a finite number of numbers or even all but one number. Here is one (not mentioned in the article) *Every number is the sum of at most 8 cubes except 23 which needs 9.* This chapter was about such sequences. One point of interest is that it's hard to define rigorously.

Some Diophantine Recreations by David Singmaster. This is actually a nice history of Diophantine problems. Here is an old one that seems to recur alot: Three sisters sell apples in the marketplace. The oldest one sells 50, the middle one sells 40, the youngest one sells 10. They each bring home the same amount of money. How is that possible? Well, they each sold the apples at two different prices (one price in the morning, and a lower price in the afternoon when they wanted to make sure they cleared their stock). How much did they charge and how much did they bring home? There may be several answers so other constraints may be given. You can replace the three sisters with any number of sisters and the numbers 50, 40, 10 with other numbers.

Who Wins Misere Hex? by Jeffery Lagarias and Daniel Sleator. Hex is a well known game where it is know that player I wins. There is a parameter n- the size of the board. Misere Hex is (as is usual for Misere games) reverses the criteria for who wins and loses. This chapter solves the game completely! Player I wins iff n is even. They also show (it is part of the proof) that whoever loses can force the game to be played to the bitter end.

How Random are $3x+1$ *Iterates?* by Jeffery Lagarias. Consider the function (1) $f(x) = x/2$ if x is even, (2) $f(x) = 3x + 1$ if x is odd. The *Collatz Conjecture* states that for all $x \in N$ the sequence $f(x), f(f(x)), \ldots$ will eventually reach 1. (Note that $f(1) = 4$, $f(4) = 2$, $f(2) = 1$ so the sequence will be periodic.) This is still open and seems hard. Jefferey Lagarias is an expert on the problem (see his website of papers on it). In this paper he looks at estimations of when the sequence gets to 1.

3 Puzzlers' Tribute

The Toast Tributes This section has many chapters that are toasts to people who have recently passed away. Some are biographical, some are problem sets. All are touching.

Tantalizing Appetizers: Challenges for the Reader.

A Clock Puzzle by Andy Latto. There is a clock where the hour hand and the minute hand are the same length. Note that I can usually still tell what time it is. For example if one hand is on the 12:00 and the other on the 3:00 then it has to be 3:00 and not 12:15, since at 12:15 the hour hand would not be exactly on the 12:00. How many times of the day can I NOT tell what time it is?

Six Off-Beat Chess Problems by John Beasley. Usually in a chess problem it is not important to know what the *prior* move was. For all six of these problems you can deduce something about the prior move, and what you deduce is important. These problems require more cleverness than chess ability. And they are all fun! (Solutions are included).

Smoked Ham: A Course in Magic I am not sure what the title of this section means. However, the articles in it are about magic. Most of them are about cards, dice, and knots. One is a nice poem *Casey at the Fox* (The Fox is a magic show) about a magician who disappoints.

Chef's Caprice This section does not have much of a theme. One interesting article in it was *Coincidences* by A. Ross Ecker who investigates using math whether some phenomena is a con incidence or not (he admits the term is not that well defined). For example, Abraham Lincoln and Charles Darwin were born on the same day. Is that a coincidence? What is the probability that two famous people will be born on the same day? He deduces that the probability is high enough that we should not be surprised it happened.

Wild Games (and puzzles)

Early Japanese Export Puzzles: 1860's to 1960's by Jerry Slocum and Rik van Grol is a historical review of such puzzles. It is interesting though there is not much math in it. *The Partridge Puzzle* by Robert Wainwright is the following: Recall that

$$1 \cdot 1^2 + 2 \cdot 2^2 + 3 \cdot 3^2 + \cdots + i \cdot i^2 = N^2$$

where $N = \frac{i(i+1)}{2}$. Hence you might be able to use one 1×1 tile, two 2×2 tiles, ..., i $i \times i$ tiles to tile the $N \times N$ grid. For which i can you do this? For $i = 2, 3, 4, 5$ this is not possible. For $i = 11$ such a tiling is possible. The cases of $i = 6, 7, 8, 9, 10$ are unknown. The article discusses all of this and variants.

Mathematical Entrees and Mathematical Treats (two different but similar sections)

Fermat's Last Theorem and the Fourth Dimension By Jim Propp gives a nice history of FLT including Wiles result. It is probably the most sophisticated mathematical article in all 6 books. *Games People Don't Play* by Peter Winkler was an excellent series of games. I originally thought people didn't play them because they weren't real games (most uses of the word *game* in mathematics are not really fun- see my blog entry `http://blog.computationalcomplexity.org/2010/06/whats-your-game-mr-bond-sequel.html`). But actually these are games where one of the player has a an unfair advantage, which is why they are not played. Here is an example: Paula writes down two integers on two different pieces of paper. They must be different. Victor picks one of the pieces and looks at it. He then bets \$1.00 that either the number he has seen is bigger or smaller than the number he has not seen. Does Victor have a strategy that is better than 1/2 probability of being right. Surprisingly yes. *Some Tricks and Puzzles* by Raymond Smullyan contains his usual brand of paradoxes. A nice light read. *How Recreational Mathematics can Save the World* by Keith Devlin is about how applied pure math can be (or perhaps how pure applied math can be). He does end up with some real applications- of hard math. In the end, alas, Recreational math cannot save the world. *Sum Free Games* by Frank Harary are games where players pick numbers and either hope to achieve or avoid being the one who causes there to be 3 numbers x, y, z where $x + y = z$. This kind of game has its roots in Ramsey Theory.

4 Tribute to a Mathemagician

This books section titles are **In Memoriam**, **Braintreasures**, **Brainticklers**, **Brainteasers**, **Braintempters**, **Braintauters**, **Braintools**. I will dispense with telling you which chapters I describe came from which section. Telling the different between a Braintempter and a Braintauter is an unsolved Brainteaser.

The Incredible Swimmer Puzzle by Stewart Coffin is a charming story about the puzzle and includes the puzzle itself. Apparently Stewart made up the puzzle but it had a typo and this lead to confusion that was finally straighted out after about 40 years. *A History of the Ten-Square* by A. Ross Eckler is about the quest for a 10 by 10 grid of letters so that every row and column is a word. Or Phrase. Or something. They give evidence that if you use a standard dictionary you will not succeed. However, they manage with larger sources of words such as peoples names or places. Lower order squares were known and found by hand; however, the 10-square was only possible with modern computers.

Underspecified Puzzles by David Wolfe and Susan Hirshberg are really really underspecified. Fortunately solutions were provided and were amusing. *Five Algorithmic Puzzles* by Peter Winkler is, as the title says, Five Algorithmic Puzzles. Most involved a parameter that was not obvious that you had to note was always increasing. *Upstart Puzzles* by Dennis Shasha is about puzzles where the originator had a solution in mind for the original puzzle, which is correct, but more advanced forms the originator does not know the solution. In some cases nobody does.

The Burnside Di-Lemma: Combinatorics and Puzzle Symmetry by Nick Baxter blurs the line between recreational and serious mathematics. Combinatorics in general does that. Usually recreational mathematics uses simple tools. The Burnside Lemma is not a simple tool. However, this article presents it well. For those who do not know, Burnside's Lemma uses group theory to count objects that have unusual symmetries. The classic example is counting the symmetries of a cube that has 3 sides RED and 3 sides BLUE (it matters where the sides are).

5 Mathematical Wizardry

The Ig Nobel Prizes by Stanley Eigen is about the Ig Nobel Prize which goes to science that makes us laugh but also think. The criteria is somewhat loose in that it goes to false science (e.g., Calculating the odds that Mikhail Gorbachev is the Anti-Christ) and to funny but valid science (e.g., an experiment involving a black tar in a funnel which outputs one drop every nine years.)

Packing Equal Circles in a Square by Peter Gabor Szabo talks about the following problem: We want to put n overlapping identical circles into a square of side 1. What is the max radius? For $1 \le n \le 30$ the answers are known. Some use some math of interest and some use computer work. Some of the numbers are nice ($r_4 = 0.25$) but some are nasty $r_3 = \frac{8-5\sqrt{2}+4\sqrt{3}-3\sqrt{6}}{2}$. r_{11} is far nastier in that it involves square roots of linear combinations of square roots; however, r_{19} is even worse since we only know that it is the root of a particular polynomial of degree 10. This chapter tells us some information but also has good references. This is appropriate since this material involves serious math.

Uncountable Sets and an Infinite Real Number Game by Matthew Baker gives an alternative proof that the reals are uncountable via games. He also uses games to prove other theorems in analysis.

The Association Method for Solving Certain Coin-Weighting Problems by Dick Hess gives a *general method* that works to solve several coin weighting problems. The classic version, 12 coins one is counterfeit (heavier or lighter but we don't know which) and a balance scale, falls out of his methods. He looks at different numbers-of-coins and different types of weightings.

6 A Lifetime of Puzzles

The first section has four articles on magic. Surprisingly (at least to me) is that one of these articles had serious mathematical content. Diaconis and Graham have an article *Products of Universal Cycles* where they look at generalizations of de Bruijn cycles and apply them to magic. A k-ary de Bruijn cycle (also called a de Bruijn sequence) is a sequence of 0's and 1's such that each window of width k shows a different element of $\{0,1\}^k$. For example 10111000 is a 3-ary de Bruijn sequence. How should this be defined if you have more than 2 letters in your alphabet? You might think that if your alphabet is $\Sigma = \{0,1,2,4\}$ and $k = 3$ you want different elements of Σ^3. Not quite. You want different orderings. For example, if you have already seen 124 you do not want to see another 3-sequence that is increasing. As an example 132134 is a 3-ary de Bruijn cycle. These object seem to be very interesting mathematically and apply to Magic.

Can Ants be interesting mathematically? Peter Winkler thinks so. He has a chapter *The Adventures of Aunt Alice* that is a sequence of problems involving ants. Here is one: *Twenty-five ants are placed randomly on a meter-long rod, oriented east-west. The thirteenth ant from the west end of the rod is our friend Aunt Alice. Each ant is facing east or est with equal probability. They proceed to march forward (that is, in whatever direction they are facing) at 1 cm/sec; whenever to ants collide, they reverse directions. How long does it take before we can be certain that Alice is off the rod?* There are 10 puzzles of this type in the chapter, with the answers. They are of the type that once you see the answer it is obvious but before that it might not be.

7 Homage to a Pied Puzzler

Martin Gardner called Sam Loyd *America's Greatest Puzzlist.* Loyd is credited with many puzzles and indeed deserves credit for ... some of them. Jerry Slocum in *Sam Loyd's Most Successful Hoax* gives strong evidence that Sam Loyd did not invent the 15-puzzle which is often attributed to him. This is not just people attributing the puzzle to Sam Loyd after the fact. Sam Loyd himself claimed to have invented it. I am not quite sure what to make of it. On the one hand, it is immoral to claim to be a puzzle's inventor when you are not. On the other hand, this is a a Hoax, and that is what puzzlers do.

There are seven articles in this book with the word *seven* in the title. I doubt this is a coincidence. One of them is *Seven Staggering Sequences* by Neil Sloane. I describe one that is truly staggering. Let $b(n)$ be defined as follows:

- $b(1) = 1$.
- To find $b(n+1)$ you write down
  $$b(1), b(2), \ldots, b(n)$$
  and find an X, Y sequences of naturals, $k \in N$ such that
  $$b(1), b(2), \ldots, b(n) = XY^k$$
  and k is as large as possible. Let $b(n+1) = k$.

We work out the first few.

1. $b(1) = 1$

2. $b(2) = 1$ using X is the empty string, $Y = 1$, and $k = 1$.

3. Look at the sequence $b(1), b(2) = 1, 1$. Let X be the empty string, $Y = 1$, and $k = 2$. So $b(3) = 2$.

4. Look at the sequence $b(1), b(2), b(3) = 1, 1, 2$. Let $X = 1, 1$, $Y = 2$, $k = 1$, So $b(4) = 1$.

The first few terms are

$$1, 1, 2, 1, 1, 2, 2, 2, 3, 1, 1, 2, 1, 1, 2, 2, 2, 3, 2, 1, 1, 2$$

Does 4 ever appear? Yes, $b(220) = 4$. Does 5 ever appear? Yes. They proved (in a different article) that there is some $x < 10^{10^{23}}$ such that $b(x) = 5$. More generally they prove that you get x within roughly $2^{2^{3^{\cdots m}}}$.

Only one word describes such a sequences: Staggering!

8 Opinion

There are chapters on a large range of topics: Magic, History, Recreational Mathematics, Serious Mathematics, and some articles that are hard to classify. Clearly the line between recreational and serious mathematics is hard to draw; however, in this book, the line between magic and math of any sort is also hard to draw.

Even though there is a large range of topics they all have a certain spirit. They are all fun! (The articles in Memoriam may be an exception. Or not.) They are all by authors who love their subject and want to share it with you. And, as noted 6 pages ago, they are well written.

How can you, my readers who are somewhat sophisticated mathematically, use these books?

- Many of the chapters can be the basis for class projects on a variety of levels.

- The math presented in these books is pretty easy but the pointers to other papers and open problems is at times deeper.

- There is so much variety in these books that I suspect there are mathematical things in them that you simply did not know and can learn. So do so!

Review of

Comprehensive Mathematics for Computer Scientists 1 [3]

Author: Guerino B. Mazzola, Gerard Milmeister, Jody Weissmann
Publisher: Springer, 2006
ISBN 3540368736, $52.95
Available online at `http://coreitem.umn.edu/coreitem/`

Reviewer: Sage LaTorra

1 Introduction

Comprehensive Mathematics for Computer Scientists 1 is perhaps one of the most descriptive titles I've ever encountered for a book. Mazzola, Milmeister and Weissmann intend their book to be an overview of all the background an aspiring theorist would need to venture into the field, and for the most part they succeed at that goal.

The book takes a ground-up approach, starting with propositional logic and building a swerving path that hits the most important information in a number of areas. Most topics are dealt with and moved past with determined brevity, the text therefore being composed primarily of definitions, theorems and proofs strung together with only brief discussion. Graphics are spread through each topic, breaking the monotony of Greek characters and short proofs.

The reviewed volume (part 1 of 2) covers the basic theory of sets and numbers, algebra, graphs, automata, and linear geometry. The coverage is intentionally broad rather than deep, aimed at college level students. The choice of material and intent of the book make it most appropriate for a graduate-level introduction to theory, but none of the material is so advanced it couldn't potentially be taught to advanced undergraduate students. The target audience is likely students with a working knowledge of applied computer science looking for a foundation for further theory courses. The table of contents accurately summarizes the material within, for any instructors wanting to compare the list of topics to those topics they intend to cover.

2 Review

Considered as an overview of pertinent facts to basic theoretic computer science, Comprehensive Mathematics for Computer Scientists 1 is a success. The ground-up approach forms a narrative of sorts, allowing an attentive reader to gain further enjoyment from seeing certain ideas appear repeatedly. This narrative also makes an effective (and commonly used) blueprint for courses of this sort in many computer science departments.

Each topic area is given just enough space to cover basic concepts without diving into more specific ideas that may not be pertinent to every student or reader. A student who absorbs this book should be able to move into an advanced course in any area covered with no trouble at all.

As a textbook, accuracy is of particular value here. Thankfully this is a second edition and showed the quality one would expect from a revised volume. Proofs are accurate and if some step is omitted (due to complexity or space) it is clearly mentioned. There are a few grammatical missteps, but they are minor and should not interfere with reading at all.

However, taken as a teaching tool, the review is less favorable. The dense text, almost devoid of sections mentioning practical applications, means that only the most dedicated students will likely be able to consume much of the information. With the exception of a section on relational databases, the only mentions of applications of this knowledge are found in the advanced topic sections. The focus of the book is obviously on theory, but some mention of ways in which this theory is important would greatly improve the odds of students actually engaging the material. As it stands, the text will be intimidating to most students, leaving only the most dedicated to find the interesting sections covering some theory behind relational databases, RSA encryption and RS error correction codes.

An instructor faced with teaching a course based on Comprehensive Mathematics for Computer Scientists 1 for the first time will also be faced with a paucity of usable exercises[4]. What exercises are included are scattered throughout the text, not gathered cleanly at the end of chapters or sections. This is far from a fatal flaw, but an instructor looking to quickly cull some questions from the text will be flipping many pages.

The quality of the exercises is also varied. The difficulty ranges between straightforward applications of simple processes (computing the power set of a set) to relatively complex proofs (at least a few proofs throughout the text are left as exercises). The authors of any text should not be relied on to effectively plan a course based on their text, but the exercises here are particularly bad for a potential instructor.

What value a potential reader can derive from Comprehensive Mathematics for Computer Scientists 1 is ultimately dependent on their use. Instructors looking to use this book as the basis for an overview of the mathematics a computer scientist needs should carefully evaluate what they need from their text, and the amount of it they expect students to absorb. However, viewed as a reference or refresher, the book is a solid choice. I would think this text would be particularly useful to computer scientists who have not studied theory in some time and need a quick refresher for some reason.

[4] Author Comment: Concerning the rare exercises/examples, the total of over 700 pages for the 2 volumes was a reason for not including more material.

Review[5] of

Creative Mathematics
By H.S. Wall
224 Pages, Hardcover, $52.95
MAA, 2006

Review by Jason Dyer Jason.Dyer@tusd1.org

> I do not state a theorem and then proceed to prove it myself. Instead, I try to get students started creating mathematics for themselves as piano teachers start their students creating music – not by lecturing to them but starting at once to develop coordination of mind and muscle. In mathematics this means training the mind to coordinate the right ideas from a set of axioms and definitions and arriving by logical reasoning at a proof or theorem. – from *Creative Mathematics*, H. S. Wall

The above quote sets the tone for a work originally published in 1963 and updated in 2006, setting out what's now known as The Moore Method or The Texas Method of teaching. In short, it is a textbook of calculus, but that descriptor fails to capture the radical vision: it emphasizes mathematical discovery, giving the background and definitions required for a student (starting with the axioms of numbers) to prove his or her way through many of the theorems of calculus.

Note the word *most*, and not *all*. The Moore Method in practice necessarily involves some cutting, because requiring the students to prove (nearly) every theorem requires some judicious selection. For example, comprehensive solving of integrals is de-emphasized (removing, as a specific example, most of trigonometry) in order to make room for a path of solving leading to computation formulas, simple surfaces, and linear spaces.

Excluding the table and glossary, the book is only 185 pages. Even at that slim length it's still difficult to say any student would make through the front to back cover in a semester, but the Mathematical Association of America has billed this new edition as "supplementary classroom material", not as a full textbook. The book is compact enough that even a few pages can provide material for an entire classroom session.

While the front matter gives the impression that all mathematics used will be proven by the students, the book is unfortunately not entirely as billed. Quite a few theorems are fully proven in the text. In some portions the book is hard to distinguish between a standard one which switches between proofs and exercises.

Given that, is the book still relevant nearly 50 years later? The gauntlet of starting from sparse beginnings and leading by proof through the valley of mathematics was taken by later authors, some with a clearer writing style (see, for example, Tao's *Analysis I*). The dense style makes it difficult for a student not already expert in the material (that is, one who didn't need the book in the first place) to use *Creative Mathematics* for self-study.

The other dated aspect is a lack of inductive reasoning. While there are moments when induction is implied ("Try to invent a definition of length of a simple graph.") it is very de-emphasized. While inductive reasoning fell out of favor for a time in the 20th century, with modern computers many find it essential to mathematical discovery. Having the exact definitions needed to answer a set of questions is restrictive compared with the vast array of thinking styles in real mathematics; as a model for teaching mathematicians to create *new* mathematics Wall's text falls short.

Still, *Creative Mathematics* is useful as more than a historical curiosity. Anyone who has a taught a Moore Method class knows selecting the right sequence to lead students can be much more difficult than it appears, so even if is now more useful as a supplemental rather than a direct resource it can provide an invaluable roadmap for mathematical concepts.

Review[6] of
Nonlinear Integer Programming
by Duan Li and Xiaoling Sun
Springer Science+Business Media, LLC, 2006
435 pages, $154.00, HARDCOVER

Review by
Justin Melvin[7]
Georgia Institute of Technology, Atlanta, Georgia

1 Introduction

The field of operations research often tackles important real-world problems that do not have provably efficient general algorithms to compute solutions, but for which finding a good enough solution or even a feasible solution is critical. In many applications, such as resource allocation, portfolio selection, redundancy optimization in reliability networks, and chemical engineering, many important problems have a nonlinear structure where some component of the problem represents a binary decision or an indivisible quantity, a class of problems called nonlinear integer programming (NLP) problems. The field of integer *linear* programming is already NP-hard, so it is not likely that there is ever going to be a generic efficient algorithm found for the these problems, and the introduction of *nonlinearity* further complicates the issue. Despite the lack of optimal polynomial time algorithms for the general case, there has been a great deal of work in the development of effective heuristics, the identification and exploitation of special subproblem and substructures, and the performance of large-scale experiments of various algorithms and parameters on real-world problems of this variety. *Nonlinear Integer Programming* is an introduction in the state-of-the-art in these methods for solving discrete optimization problems under nonlinear constraints.

This book follows the general structure of introducing a special case of a problem class for which good heuristics and strong guarantees exist and moving into more and more general problems. Where possible, the authors derive theoretical results and provide experimental data. As one would expect, the techniques and experimental results grow weaker as the problems become more general and lose any special structures to exploit.

2 Summary

2.1 Introduction

The book begins by motivating the general field of nonlinear integer programming and a discussion of the inherent difficulties of solving such problems; then, the subsequent chapters start with specific classes of problems with nice structures which evolve into heuristics for more and more general classes.

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2.2 Optimality, Relaxation and General Solution Procedures

The second chapter of the book gives a crash course in discrete and convex optimization techniques, including branch-and-bound, partial enumeration and Lagrangian/continuous relaxations. Then, the authors describe some basic results in continuous and discrete convex optimization, including a discussion of integrality gaps, separable convex problems, and finally 0-1 quadratic programming.

2.3 Lagrangian Duality Theory

This chapter provides an overview of properties of the Lagrangian relaxation, which incorporates hard constraints into the objective function. The chapter covers multiple aspects of the Lagrangian dual, including techniques for solving the dual problem, criteria for the strong duality, the effects of perturbations, and specific applications to 0-1 quadratic optimization.

2.4 Surrogate Duality Theory

While the Lagrangian relaxation replaces difficult constraints with additional terms in the objective function, the surrogate dual replaces multiple constraints with a single constraint. This chapter covers techniques for producing surrogate constraints and techniques to produce good solutions to the dual problem.

2.5 Nonlinear Lagrangian and Strong Duality

This chapter covers nonlinear reformulation techniques to enhance the Lagrangian formulation and achieve strong duality. This includes convexification by the p-power convexification, logarithmic-exponential dial reformulation, and an in-depth study of single-constrained nonlinear integer programs. The chapter includes numerous contour plots demonstrating the relative strength of the various reformulation techniques.

2.6 Nonlinear Knapsack Problems

The linear knapsack problem is a classical problem in operations research, and in this chapter, which is the strongest in the book, the authors investigate the extension of the knapsack problem to nonlinear constraints. The techniques discussed in this chapter are the basis for many of the results in the remainder of the book. This chapter is replete with examples, algorithms, proofs, and diagrams and approaches the problem from several directions: branch-and-bound approaches, Lagrangian and domain cut methods, and 0-1 linearization. The chapter deals separately with the special case of concave nonlinear knapsack problems and finally an extended case study on reliability optimization of series-parallel graphs.

2.7 Separable Integer Programming, Nonlinear IP with Quadratic Objective Function, Nonseparable Integer Programming

This triplet of chapters deals with increasingly more general problems: first separable integer programming which may be solved through dynamic programming and Lagrangian decomposition; then it discusses the general case with a quadratic objective, for which there are contour cut methods and extensions of Lagrangian relaxations; and finally it approaches the nonseparable case, which lacks the special structure of the previous two chapters and must be approached with a technique like branch-and-bound, Lagrangian decomposition without strong duality guarantees, and convexification. Each of these chapters provides a computational study comparing the various methods.

2.8 Unconstrained Polynomial 0-1 Optimization

This chapter describes extensions of unconstrained quadratic 0-1 optimization to the general unconstrained 0-1 polynomial case, including roof duality, a local search algorithm, convex relaxations, and an in-depth description of the quadratic 0-1 case with applications to the maximum-cut problem.

2.9 Constrained Polynomial 0-1 Optimization

This chapter applies heuristics used in linear integer programming, such as cutting plane algorithms and branch and bound, to the constrained polynomial 0-1 programming case. It includes techniques for linearizing the problem and form converting the problem into an unconstrained optimization problem. The chapter concludes with a study of the quadratic 0-1 knapsack problem and a comparison of various heuristics.

2.10 Two Level Methods for Constrained Polynomial 0-1 Programming

This chapter describes alternative solution methods for constrained polynomial 0-1 programming, including Taha's method of replacing monomials with decision variables, convergent Lagrangian methods with objective level cuts, and p-norm surrogate constraints. The chapter concludes with computational studies of each method on randomly generated test problems.

2.11 Mixed-integer Nonlinear Programming

Finally, this chapter describes methods for mixed-integer nonlinear programming, where some variables are continuous and others are discrete. The techniques described are similar to those used in the mixed-integer linear programming case, such as branch-and-bound, the general Benders decomposition, outer approximation techniques, and the convexification of nonconvex MINLP instances. This chapter includes some examples, but does not provide a computational study of the various techniques.

2.12 Global Descent Methods

The final chapter studies the difficult problem of optimization over a finite integer set of a noncontinuous objective function. Since there is almost no structure to exploit in these problems, the algorithms and heuristics present in this chapter are general. The chapter discusses local search versus global descent in the abstract, and then investigates global optimum properties of a class of two-parameter discrete global descent functions. Finally, the chapter gives a generic algorithm for the discrete global descent method and provides some computational results on linear integer programs, quadratic integer programs, and some box constrained/unconstrained integer programming problems.

3 Opinion

This book is written at the level of a professional researcher or practitioner in optimization and graduate students or advanced undergraduates who are already well-acquainted with basic optimization techniques in mathematical programming, particularly in the formulation of problems as nonlinear or integer programs. While many of the algorithms and heuristics presented in the book are self-contained, they are often not motivated by real-world examples outside of the introductory chapter. As such, the reader must have a good level of mathematical sophistication in order to follow the proofs and make use of the book.

The book also contains flow charts and descriptions of algorithms implementing the ideas discussed in the mathematical exposition, so the reader should also have some basic experience with computer science and the implementation of algorithms.

Overall, this is a good exposition of the state-of-the-art in finding solutions to generally intractable but important problems.

Review of
Complex Social Networks [8]
Author: Fernando Vega-Redondo
Publisher: Cambridge University Press, 2007
978-0-521-67409-6, $37.99
Econometric Society Monographs

Reviewer: Carl Kingsford (carlk@cs.umd.edu)

1 Overview

This is a review, written by a computational biologist, for a CS theory newsletter, about a book in an economics monograph series on the subject of complex social networks. This confluence of disciplines illustrates how the subject of the reviewed book has become central to many fields. Although supported by a long history of study in sociology, graph theory, and computer science, the science of complex networks has recently become more relevant as networks have become more commonplace in both scientific pursuits and everyday life.

This book considers random networks generated under various models as idealized approximations to real-world networks. It seeks to understand how network structure affects the propagation of attributes and the decision-making processes of the entities modeled as vertices. The viewpoint taken is p©robabilistic and statistical, and the book focuses on ensembles of random graphs rather than particular instantiations of networks. Many of the questions the book addresses can be phrased as determining when a given process (e.g. the spread of a disease or the adoption of a technology) will affect a significant fraction of the network with high probability. In particular, the book develops and reviews the phenomena of diffusion, search, and "play" (the latter being the situation where nodes engage in actions that are constrained or informed by the network structure, often in a game-theoretic setting) in a number of contexts.

2 Summary of Contents

Chapter 1 ("Introduction") gives several motivating application domains where networks have been measured empirically (e.g. citation, email, neural, and friendship networks). The chapter also describes the importance of network effects in socioeconomic situations such as efficiency of labor markets, adoption of new technologies, recruitment to large social movements (such as political revolutions), organization of R&D partnerships between companies, structure of informal insurance relationships to mitigate risk, and the effect of peers on criminal behavior. While none of these settings are explored in depth, copious relevant citations are provided for the interested reader to explore the topics in more depth.

Chapter 2 ("Complex Networks: Basic Theory") is an excellent introduction to many of the fundamental results in the study of complex random networks. The Erdös-Rényi, configuration, small-world, and preferential attachment models of random graphs are introduced in a rigorous way, as are the properties of clustering, cohesiveness, and betweenness. Conditions that govern the emergence of a large connected component, small diameter, or high clustering coefficient are derived.

Chapters 3, 4, 5 and 6 contain the main, advanced technical material of the monograph. Chapter 3 ("Epidemic Diffusion") considers the spread of information or disease through a network. The chapter explores the well-known SIR and SIS epidemiological models. SIR idealizes the case where the effects of diffusion are permanent: nodes transition from Susceptible to Infected to Recovered states. Infected nodes infect their susceptible neighbors with some probability, and they recover to become forever immune at a given rate. Here the question is what rates of infection and recovery are needed to make it likely that a large fraction of the network becomes infected? The SIS model (where nodes transition from Susceptible to Infected to Susceptible states) idealizes the case where the effect of infection on a node is temporary. In this case, the main question is under what conditions does the infection not die out? The chapter also investigates an interesting twist on the standard models: what happens if nodes break down and become unable to propagate the infection with some probability?

Chapter 4 ("Neighborhood Effects in Diffusion and Play") extends the models of Chapter 3 to the setting where the fraction of a node's neighbors that adopt the diffusing property determines whether the node itself adopts the diffusing property. This is further extended to the case where each node adopts a state based on a payoff matrix describing a game played with each of its neighbors.

Chapter 5 ("Search in Social Networks") turns to search in random networks. When a node receives a request, how can it forward the request to a node that can answer it if it only knows local information about its neighborhood? These routing algorithms are considered when only topology is available, and also when the nodes have "locations" (or sets of properties) that are derived from several kinds of models. The problem of congestion (workload at a node that delays the processing of a search) is also considered in some depth.

Chapter 6 ("Search, Diffusion, and Play in Coevolving Networks"') looks at several of the topics of the previous chapters in the case where the network structure depends on the process of search, diffusion, or play. In other words, it examines the case where the actions of the vertices now affect the existence or non-existence of links (the network "coevolves" with the processes occurring on the network). The chapter considers models of network growth where entering nodes execute a search to find neighbors to which to link. It also considers evolving networks where nodes regularly update their strategies in a coordination game to maximize their payoff in a situation where edges are created with higher probability between two nodes if the nodes have chosen coordinated strategies.

In the above summary, for the sake of space, I have simplified and omitted a number of the interesting topics (e.g. vaccination strategies in chapter 3, designing networks to reduce congestion in chapter 5) that make the book so comprehensive a treatment of complex network theory.

3 Style

The text is approachable and requires only an undergraduate background in probability theory and a sufficient amount of the proverbial mathematical maturity. Many of the arguments employ generating functions and mean field theory, and these techniques are reviewed in thoughtful appendices. The main text is readable, clear, and direct, but completeness is not sacrificed for this simplicity: derivations of nearly all results are provided, along with sufficient intuition to understand them. Although there is a tremendous amount of detail in its 294 pages, the exposition is never dense or hurried. One minor criticism: the format does not always make it easy to skim the chapters to determine the results and the assumptions on which they depend. The text is not presented in a Theorem-Proof structure, which makes the initial reading far more enjoyable, but hinders somewhat the monograph's use as a quick reference.

4 Opinion

The book was engaging to read. The author seems to have paid an unusual amount of attention to presenting the various subjects so that their commonalities become apparent, and as a result the book feels cohesive. Despite the title including the word "social," the theory developed here is quite general and ought to be of interest to researchers working in biological and technological networks as well.

The book is probably too difficult for all except the most precocious undergraduate students, but the material is perfectly suitable for mathematically inclined graduate students in computer science. This would make an excellent textbook for a graduate class on complex networks, particularly if it were supplemented with a few papers dealing with some of the algorithmic aspects in more detail.

Among the several books that I have read that purport to survey the theory of complex networks from a probabilistic perspective, this is by far the best, and I recommend it highly.

Review of[9]
The Calculus of Computation:
Decision Procedures with Applications to Verification
by Aaron R. Bradley and Zohar Manna
Springer, 2007, 366 pages, Hardcover, $50.00

Review by
Carlo A. Furia caf@inf.ethz.ch

1 Main Themes

The book covers three main areas, tightly interconnected: mathematical logic, proofs of correctness for sequential programs, and decision procedures for a few decidable logic theories.

The introduction to mathematical logic covers the basics of propositional logic, first-order logic, and logical theories. The latter are essentially domain-specific logical frameworks using first-order formulae to capture the features of the domain under consideration.

The correctness of a program can only be proved against a *formal specification* of its intended behavior. A formal specification usually consists of pre and postconditions, loop invariants and other intermediate assertions, and ranking functions to reason about termination. The book shows how to annotate a program systematically with first-order formulae, usually cast in a logical theory suitable for the data types the program operates on. Once a program is adequately annotated, verifying its correctness reduces to proving the validity of a few formulae called *verification conditions*, derived automatically from the annotations.

A decision procedure is an algorithm to decide the validity of logic formulae in a given theory (or fragment thereof). If the verification conditions for a program are expressible in a fragment for which a decision procedure is available, proving the correctness of the annotated program can be fully automated. Correspondingly, the book focuses on several logical theories for which (usually efficient) decision procedures exist, and which can express the verification conditions occurring in practice in programs.

While the book does not deal with the low-level details relevant to the well-engineered implementation of decision procedures, it refers to $\pi\mathsf{VC}$, a publicly available system that handles annotated programs (written in a simplified C-like language), generates verification conditions, and verifies them automatically with a number of decision procedures included in the system's back-end.

2 Summary

The book comprises two parts. The first part (Chapters 1 through 6) deals with mathematical logic and its application to specify and reason about sequential programs. The second part (Chapters 7 through 13) presents several decision procedures for interesting fragments of logical theories, and includes references to further reading.

Chapter 1 presents propositional logic. It includes the standard definitions of syntax and semantics, as well as how to reason about satisfiability and validity using truth tables and formal proofs (called *semantic arguments*). The rest of the chapter revisits these basic techniques with a more "algorithmic" stance, which leads to a presentation of the widely used algorithms of *resolution* and *DPLL* (invented by Davis, Putnam, Logemann, and Loveland). These are at the core of every modern implementation of a solver for propositional satisfiability.

Chapter 2 introduces the syntax and semantics of first-order logic. The presentation highlights the connections with the material from the previous chapter about propositional logic and shows how first-order logic generalizes it (for example, the semantic argument generalizes to reason about the validity of first-order logic formulae). The second part of the chapter deals with more advanced topics: some notes on decidability and complexity, and some technical theorems about soundness, completeness, and other meta-theorems such as the Compactness theorem and the Craig interpolation lemma. These advanced results are marked as optional and are used later in the book only to prove the completeness of some techniques that combine decision procedures.

Chapter 3 shows how first-order formulae can constrain the semantics of first-order logic and build domain-specific logical frameworks called *first-order theories*. After a general introduction on the topic, the chapter presents and illustrates a number of well-known first-order theories that are often relevant to the formal analysis of programs. First, mathematical theories of general usage are considered: the theory of equality with uninterpreted functions, the theories of Peano and Presburger arithmetic for integer numbers, and the theories of the real and rational numbers. Then, a few theories to reason about programs operating on certain data structures are discussed: a theory of lists *à la* Lisp (with several variations) and a theory of arrays.

The application of first-order theories to reasoning about programs requires one more ingredient, supplied by **Chapter 4** which presents *induction techniques*. These include the standard stepwise induction from arithmetic, as well as more advanced incarnations such as well-founded induction (which underpins proofs of termination) and structural induction (useful to prove meta-results about formulae and formal notations in general).

Chapter 5 focuses on sequential *programs* and how to reason formally about their correctness. First, it introduces pi, a simple imperative language that supports annotations in the form of full-fledged first-order logic formulae. The chapter next discusses the role of *annotations* as specification of a program's behavior, and illustrates it on several example programs of varied complexity. The *inductive assertion method* is chosen to illustrate the notion of *program correctness*. To verify the correctness of a program with the inductive assertion method, extract a number of finite loop-free paths from the control-flow graph of a program and annotate each path with a detailed pre and postcondition. Then, generate automatically a set of *verification conditions* for each annotated path with the machinery of the weakest-precondition calculus. If each first-order formula appearing as verification condition is valid within its logical theory, then the program is correct. The chapter finally extends the inductive assertion method with the notion of *ranking functions* to verify the *termination* of programs as well.

The description of the inductive assertion method in Chapter 5 assumes the availability of sufficiently detailed assertions that precisely capture the semantics of the program and that are amenable to inductive proofs. How to come up with such assertions in the first place is the object of **Chapter 6**, which illustrates a number of *heuristics to synthesize relevant assertions* documenting common programming patterns. The chapter concludes with an extensive example of manual program verification with an implementation of the Quicksort algorithm, which illustrates the techniques of Chapters 5 and 6 in detail.

Chapter 7 presents two *decision procedures* for two *theories of numbers* that occur often in practice: the theory of arithmetic over the integers without multiplication (usually called Presburger arithmetic) and a similar theory over rational numbers. Both theories are fully decidable; their decision procedures are presented in the same chapter because they are both based on the technique of *quantifier elimination*. As the name suggests, the technique consists of repeatedly rewriting a formula into an equivalent one with a quantifier removed, until a quantifier-free formula equivalent to the original formula is obtained; the satisfiability or validity of the quantifier-free formula can then be decided by straightforward inspection. The chapter illustrates the basic technique on several examples, introduces some optimizations for formulae with a special structure, and states the lower- and upper-bound complexity.

In spite of the optimizations that may be introduced, general quantifier elimination algorithms are quite expensive in practice. On the other hand, many of the verification conditions that appear in program verification do not require the full expressive power of arithmetic theories. **Chapter 8** thus focuses on the fragment of arithmetic formulae over the rationals without multiplication and with only existential (or, dually, only universal) quantifications over variables. The validity of formulae in this fragment can be cast as a *linear program*: the problem of maximizing a linear function of some unknowns subject to constrains defined by a system of linear inequalities. The chapter presents the well-known *simplex algorithm* to efficiently solve linear programs. It also includes a concise introduction to notions of linear algebra, which makes the chapter self-contained.

Chapter 9 considers another interesting fragment which is efficiently decidable: the theory of *equality with uninterpreted functions* with only one type of quantifier. The main interest of this fragment, from a program verification perspective, lies in the fact that it underpins many properties of recursive data structures such as Lisp-like lists and arrays. The chapter first presents the basic algorithm to efficiently decide validity in the theory of equality with uninterpreted functions with only one type of quantifier; it is essentially a *union-find* algorithm. Then, the algorithm is finessed to handle similar formulas in the theory of Lisp-like lists and in the theory of arrays.

Reasoning about program correctness often requires to handle formulae expressed in *combinations of theories*. For example, the verification conditions for a program manipulating an array with integer index are likely to require integer arithmetic as well as the theory of arrays with equality. **Chapter 10** presents the *Nelson-Oppen method*: a general technique to reason about formulae in the combination of decidable quantifier-free fragments of theories which only share a notion of equality. The chapter first presents a nondeterministic version of the Nelson-Oppen method. It is not meant for efficient implementation but is useful to illustrate the rationale of the method and to justify its correctness. Then, it illustrates an efficient deterministic version which is based on the propagation of facts about equalities among the decision procedures combined. The chapter concludes with some notes on the correctness of the Nelson-Oppen method (requiring some technicalities about first-order logic from Chapter 2) and on its complexity.

Most of the decidable fragments for which an efficient decision procedure is available only allow one type of quantifier. This restriction leads to an expressiveness which is often sufficient to formalize non-trivial verification conditions. A noticeable exception is reasoning about programs manipulating arrays, where it is often necessary to combine existential quantification (to express the existence of a certain value or index) with universal quantification (to express properties invariant over a certain range). **Chapter 11** presents a fragment of the theory of the array which allows some constrained form of mixed universal and existential quantification. The fragment is decidable with a decision procedure which replaces universal quantifications with conjunctions over a finite set of terms. The transformed formula contains only existential quantifications and is thus decidable with the techniques of Chapter 9. The fragment can be extended to include some integer arithmetic and to formalize properties of hash tables as well. The expressiveness of the fragment is also close to optimal in that most of its "natural" extensions are undecidable.

A decision procedure automates the validity check of verification conditions. Generating the verification conditions, however, requires sufficiently detailed annotations the availability of which rests, in general, on the ingenuity of the programmer. *Abstract interpretation* techniques, presented in **Chapter 12**, can alleviate this task by partially automating the extraction of annotations from the static analysis of a program text. The chapter first presents abstract interpretation as a general framework. Then, it instantiates the framework to derive annotations in the form of constant intervals (e.g., "variable x is between 3 and 100) and linear constraints (e.g., "variable x is twice as variable y"). The presentation of abstract interpretation uses a notation and terminology consistent with the rest of the book, which is quite different than the standard presentation of abstract interpretation techniques — where a more "algebraic" point of view is used. The last part of the chapter bridges this gap and presents the standard terminology of abstract interpretation frameworks.

Chapter 13 concludes the book by listing several references for further reading on varied topics in program verification, such as decision procedures, concurrency, and temporal logic reasoning.

3 Opinion

The book is an excellent introduction to program verification and an overview of automated decision procedures.

The authors did a very good job at making the book self-contained: the notation and terminology is introduced once and then used uniformly throughout the book, even when this required to adapt the "standard" presentation of a topic (see especially Chapter 12 on abstract interpretation). The introductory material on propositional and first-order logic is suitable for novices to mathematical logic, and focuses on the essentials and the practical aspects that are really necessary to reason about the correctness of programs, avoiding as much as possible technical results about mathematical logic that, while interesting, are probably best postponed to more advanced studies.

The choice of topics is convincing and the inclusion of short historical notes and bibliographical references at the end of each chapter makes it easy for the interested reader to find more material on any subject of particular relevance.

The presentation style mixes formal and informal explanations and uses plenty of examples fully worked out. This makes the book very readable and suitable for self-study.

I can think of just a few suggestions for improvement. Regarding the choice of topics, it might be interesting to include some additional material on automated theorem proving and automated assertion inference beyond abstract interpretation. Both topics have been extensively investigated in recent years, also by the authors of the book, and would make the selection of topics even more appealing. Regarding the technical content, I would expand a little the basic presentation of first-order logic, in particular of the concepts of soundness and completeness, without relegating them to the "optional" part.[10]

The book is by and large an excellent reading, both a good introduction for the novice and a useful reference for the more experienced researcher.

[10]This in turn would suggest to make more rigorous the bibliographical remarks at the end of Chapter 2, where the word "theorem" is occasionally used unorthodoxly as a synonym of "valid formula".

Review of[11] of
Algorithms on Strings
by Crochemore, Hanchart and Lecroq
Published by Cambridge 2007
392 pages, Hardcover, $60.00
Review by
Yiorgos Adamopoulos yiorgos.adamopoulos@gmail.com

1 Introduction

In my day job I am a Postmaster. Being a Postmaster involves among other things spam fighting. This basically sums up to configuring, customizing or developing software that identifies spam and deals with it. The way the technology is these days this boils down to regular expressions, Bayes or combining both. I got interested in "Algorithms on Strings" for two reasons: Understand how the "string search" part really works, and if there can be better approaches than just regular expressions and Bayes that can help me with my work.

The book deals with "stringology" that is, according to www.stringology.org, the "science on algorithms on strings and sequences. It solves such problems like exact and approximate pattern matching, searching for repetitions in various texts,... etc. There are many areas that utilize the results of the stringology (information retrieval, computer vision, computational biology, DNA processing,... etc.)".

The language of the book impressed me a lot. It is written in English plain enough to be understandable even by non native speakers and although it is full of mathematical proofs the text is not boring at all. The figures and their captions are very very helpful in understanding how the presented algorithms work, sometimes being more valuable than the text itself. I think it would not have been an easy task for me to follow the discussion without them.

Like the backcover of the book says, it is not for undergraduates. The typical undergraduate curriculum does not take one further than the first chapter. The next eight chapters all step on the first one, of which they are elaborations and more advanced treatments on specialized subjects. Chapter 5 for example presents a concise treatment on tries (and in case anyone wonders why someone outside theory is interested in tries, I refer them to the BSD TCP/IP code where PATRICIA is used).

This is not an easy book. It is a good book, but one needs pencil and paper to really follow it. Time after time I drew automata on paper in order to really "get" it. The pseudocode on the other hand is very helpful. If one does not want to follow the theory and proofs and just wants to write some code, it is fairly easy after doing some preparation to understand how the pseudocode is written. Plus the index of the book includes all the routines that are developed through it, so when one a routine is used within another and you want to find its implementation, it is quite easy. One caution though: This is not to be taken that the book is accessible to the average programmer without some formal exposure in CS theory. Automata(complete and compressed) jump all over the place and halfway through chapter 1 is the most that I remembered from my undergraduate education. To really appreciate the book and write working code one must devote time in the first chapter, and if possible do all the exercises.

2 Summary

Every chapter in the book starts with a preface of what is going to be discussed, the discussion follows, and the comes a "Notes" section. If you plan to read the book, or several chapters of it in a row, skip the prefaces (they are at most one page long). The "Notes" sections are very valuable. They provide references to every algorithm and technique discussed and pointers to the bibliography for those interested. For example the Notes of chapter 7 (alignments) explain which of the techniques are used in molecular biology which is not the typical application of string algorithms that the average programmer has in mind.

With one exception (which is an exercise) the book is self contained and builds on knowledge developed in previous chapters. Exercise 4.3, adequately named "Cheat", asks for an implementation that requires studying chapter 5 first. One cannot get away with completing the exercises without a good understanding of what was previously discussed. Even the programming exercises are brilliant, for example exercise 1.20 asks for the implementation of a bit-vector string search and then comparing that implementation to the source code of the agrep (Unix) tool. I remember browsing through this code years ago without really understanding what it did, until I did it again after chapter 1. The book also mentions briefly other Unix tools like vfgrep and diff in appropriate chapters. Exercises are designed to expand the theoretical background of each chapter. For example the Boyer-Moore automata are not part of the theory but of an exercise themselves for those who wish to dig deeper. Another great example is exercise 5.17 which asks for the detailed code of an automaton described in a figure but without pseudocode in the book.

3 Opinion

This is an excellent book, one which can be read both as a graduate textbook and used as a reference when someone needs to implement something out of the ordinary that can be covered by already available regular expression libraries. If one invests the appropriate amount of time for the first chapter, getting through the rest of the book is doable. My summer project is going to be implementing the algorithms presented in the book, which I also expect to lead to proof-of-concept implementations so that I can experiment with some ideas on combating spam, other than what is already available. Hopefully something new may come out of the time invested in the book.

Review[12] of
A View from the Top
Analysis, Combinatorics and Number Theory
by Alex Iosevich
AMS, 2007
xiii+136 pages, softcover, $30.00

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1 Introduction

This is an undergraduate-level math textbook. It is quite special because instead of specializing on a given topic, it wanders through fields as different as Calculus, Number Theory, Probability, Algebra and Geometry. The mathematics presented in it have no immediate relation to computing and algorithms, except maybe for the fact that one of the chapters is dedicated to Number theory, Probabilities and the Riemann ζ function.

2 Summary

Chapters 1–4

This book operates like a Mahler Symphony: it starts very softly, and before you realize it, you are in deep water, with drums and trumpets sounding all around you. Chapter 1 deals with Cauchy-Schwartz[13] inequality in its simplest form $\sum a_k b_k \leq \sqrt{\sum a_k^2}\sqrt{\sum b_k^2}$, proven by just a few lines of "high school math" (a much simpler way to express it is $\langle a, b\rangle \leq \|a\| \cdot \|b\|$).

This inequality is true, but the author can do better, because, as he says, "to say something interesting, one must walk on the very edge of the cliff of falsehood, yet never fall off." He gives a better version of it, the Hölder inequality: $\sum a_k b_k \leq (\sum a_k^m)^{\frac{1}{m}} (\sum b_k^{m'})^{\frac{1}{m'}}$, where m' is the "dual" of m given by $\frac{1}{m} + \frac{1}{m'} = 1$. There is also a generalization of this formula to n-term products.

In Chapter 2 the author deals with a first unexpected use of Cauchy-Schwartz inequality: take N distinct points in $\mathbb{R}^3$ and their canonical projections to the three planes, and *count the number of distinct points you obtain on the planes.* We want that number to be minimal. If you think about it, one can always choose the points in such a way that their projection on *one* plane is minimal: a single point, for example. But once you have done it for one plane, it is hard, or even impossible to reduce the number of projected points on the other planes.

Using Cauchy-Schwartz, the author quantifies this phenomenon: if S_N is the set of points, and $\#\pi_j(S_N)$ the cardinal of the j-th projection, then, however you may arrange the points in space, at least one of these numbers will always be greater than $\sqrt[3]{N^2}$ (for example, for 100 points, you will always have at least 22 points projected on one of the planes).

[13] Amusingly enough, the author breaks a record by writing the same name in three different ways on a single page (p. 4): Schwarz (the German way), Schwartz (the French way) and Schwarts (the Yiddish way).

In Chapter 3 he deals with two-dimensional projections of a set of points in $\mathbb{R}^4$. The result is that the cardinal of the set is upper bounded by the cubic root of the product of cardinals of projections on planes $(1,2)$, $(1,3)$, $(1,4)$, $(2,3)$, $(2,4)$ and $(3,4)$. To prove this he uses a very interesting interpolation result which goes as follows: let $\|a_*\|_s$ be $(\sum a_*^s)^{\frac{1}{s}}$, then if $\|a_*\|_1 \leq C_1$ and $\|a_*\|_2 \leq C_2$ then for an arbitrary $1 < s < 2$ we have $\|a_*\|_s \leq C_1^{\frac{2}{s}-1} C_2^{2-\frac{2}{s}}$.

The general case of k-dimensional projections if $\mathbb{R}^d$ has been treated by Loomis-Whitney in [4].

Chapter 4 is again about projections, but this time from a different point of view. It deals with comparisons of unit balls (balls of volume 1) and unit cubes in $\mathbb{R}^d$. For example, take the 2-ball (that's a circle). The 2-cube (that's a square) does not fit inside the 2-ball, but the projection of the 2-ball (a centered segment of length $\frac{2}{\sqrt{\pi}}$) contains the projection of the 1-cube (if this projection is parallel to a side of the square). The author asks the question: what should be the dimension of our projections for this to happen? It turns out that for an n-ball, we need to project onto a subspace of dimension $k \leq \frac{4}{\pi}(n!)^{\frac{1}{n}}$ (for $n = 2$ we do find $k = 1$). Having established that, the author delves into approximations of this formula, leading to the well-known Stirling formula.

Chapter 5

This is where we get into deep water. Given is the *incidence problem*: what is the maximum number of intersections of n lines? By simply using the Cauchy-Schwartz inequality, the author shows that it is less than $\sqrt{2}n^{\frac{3}{2}}$. He states the "sharp answer" (given in [5]): it is less than $Cn^{\frac{4}{3}}$, where C is a positive constant.

From this, the author derives quite a surprising result: take a complete graph with N vertices embedded into $\mathbb{R}^2$, and measure the lengths of edges. We are interested in the minimum number of such lengths. One would expect that using symmetry it would be easy to calculate the minimum number of distinct edge lengths of a complete planar graph.

Well, this is not the case. Using the results of the book, one can prove that the number of distinct edge lengths is larger than $C\sqrt{N}$, and the author states that the best known lower bound is CN^β with $\beta \approx .86$ (cf. [3]) and that there is a conjecture on the value $C\frac{N}{\sqrt{\log(N)}}$. Like Goldbach, this is yet another conjecture which is so easy to formulate that one can only wonder how come it hasn't been proved yet.

And here the author brilliantly jumps into a different domain of math: number theory. Take A to be a finite subset of $\mathbb{Z}$ and count the number of all possible sums $a + a'$ and products $a \cdot a'$, where $a, a' \in A$. If $\#A$ is the cardinal of A then the cardinal of such sums and products is lower bounded by $C(\#A)^{\frac{5}{4}}$.

And then, switching to yet another domain, namely graph theory, the author proves that the number of crossings of a planar graph G with n vertices and e edges, is minorated by $C\frac{e^3}{n^2}$.

If you think this is as acrobatic as it can get, you are wrong: to prove this result the author uses probability methods (!). The authors of [5] take a random subgraph G' of G where every vertex is chosen with uniform probability p: then the expectation of "surviving" vertices is np, the one of edges is ep^2, and the expected value of the crossing number is upper bounded by p^4 times the crossing number of G.

Chapters 6–8

After calculus, graph theory, number theory and probability methods, we turn now to algebra, more precisely to vector spaces over finite fields $\mathbb{F}_p$ (where p is prime). After explaining why $\mathbb{F}_p$ is a field and how we can define vector spaces and lines on it, the author presents the Besicovitch-Kakeya conjecture, which states the following: if B is a subset of a d-dimensional vector space over $\mathbb{F}_p$, big enough to contain, for a given point x, lines going through x with all possible slopes, then the size of B is lower bounded by Cp^d, where C is a positive constant. We call a point of intersection of several lines a "bush," and the conjecture says that for every such vector field there is a large bush.

Chapter 7 proves this conjecture for R^2 using (guess what!) the Cauchy-Schwartz inequality [1]. And Chapter 8 develops some of the difficulties of the high-dimensional case of the conjecture.

Chapters 9–10

After a long, although elementary in nature, introduction to probabilities (Chapter 9) the author deals with the interaction of probabilities and number theory: how can we estimate the probability that two (arbitrary) numbers are relatively prime? He easily establishes that this probability is $\frac{1}{\zeta(2)}$ (where ζ is the Riemann zeta function), and after some pages of calculus shows that the numerical value of this expressions is $\frac{6}{\pi^2}$.

Chapters 11–12

Chapter 11 is about three upper bounds: if f is a differentiable function, R a parameter and $I_f(R) = \int_a^b \exp(iRf(x))dx$, then: if f' is strictly monotonic with $f'(x) \geq 1$, we have $|I_f(R)| \leq \frac{4}{R}$ (van der Corput theorem), and if $f''(x) \geq 1$, then the inequality is a bit weaker: $|I_f(R)| \leq \frac{10}{\sqrt{R}}$.

The third upper bound is about the characteristic function χ_D of the unit disk in the plane and its Fourier transform $\widehat{\chi}_D(\xi) = \int_D \exp(-2\pi i x\xi)dx$. The author shows that $|\widehat{\chi}_D(\xi)| \leq C|\xi|^{-\frac{3}{2}}$.

In Chapter 12, the author applies the results of Chapter 11 to the following setting: let $N(t) = \#\{tD \cap \mathbb{Z}^2\}$ be the integer points of the disk of radius t in $\mathbb{R}^2$. As we know from school, the area of tD is πt^2, and this number is an approximation of $N(t)$. It happens that many mathematicians have tried to find an upper bound of the difference $E(t) = N(t) - \pi t^2$.

Once again the author brilliantly shows how close a simple textbook exercise can be to a difficult result, and even to an unsolved century-old conjecture by Hardy (the British friend of legendary mathematician Ramanujan): the upper bound $|E(t)| \leq Ct$ is given as a simple exercise, $|E(t)| \leq Ct^{\frac{2}{3}}$ is proven (Sierpinski, 1903), and $\forall\varepsilon > 0\, \exists C_\varepsilon$ such that $|E(t)| \leq C_\varepsilon t^{\frac{1}{2}+\varepsilon}$ is Hardy's conjecture. It seems that the best known proven result (Heath-Brown) is $|E(t)| \leq Ct^{\frac{19}{15}}$ and the author "conjectures" that the Hardy "Holy Grail" conjecture will be proven sometime in the 24th century. . .

Chapter 13

In Chapter 13 the author returns to finite fields $\mathbb{F}_p$ and defines the NTT (number-theoretic Fourier transform) $\widehat{f}(m) = p^{-d} \sum_{x \in \mathbb{F}_p^d} \exp(-\frac{2\pi i}{p}(-x \cdot m))f(x)$. He proves the Fourier inversion formula, as well as the Plancherel theorem saying that the sum of squares of $\widehat{f}$ (taken over all elements of $\mathbb{F}_p^d$) is p^{-d} times the sum of squares of f.

The last move of the book is to establish a connection between the Fourier transform of a subset E of $\mathbb{F}_p^d$ and the cardinal of the set of points (x, y, x', y') of E^4 such that $x + y = x' + y'$:

$$\sum_{m \in \mathbb{F}_p^d} |\widehat{E}(m)|^4 = p^{-3d} \#\{(x, y, x', y') \in E^4 \mid x + y = x' + y'\}.$$

The author shows that the cardinal of this set takes $C(\#E)^2$ as upper bound. As an example of a set satisfying this inequality, one can take $E = \{x \in \mathbb{F}_p^d \mid x_d = x_1^2 + \cdots + x_{d-1}^2\}$, in this case $C = 1$.

These results lead to the fact that if A is a subset of $\mathbb{F}_p^d$ of reasonable dimension ($\sqrt{p} \leq \#A \leq Cp^{\frac{7}{10}}$) then

$$\max\{\#(A + A), \#(A \cdot A)\} \geq \frac{\sqrt{(\#A)^3}}{\sqrt[4]{p}}$$

which is given as an exercise, and proven in a 2007 paper [2] by the author *et al.*

3 Opinion

In one of his most famous poems, Cavafy says[14] "Ithaka gave you the beautiful journey. // Without her you would not have set out. // She has nothing left to give you now. // And if you find her poor, Ithaka won't have fooled you. // Wise as you will have become, so full of experience, // you will have understood by then what these Ithakas mean." This is also the method of this book: there is no unique great result, no spectacular finale—but there is a journey. A journey through several domains of mathematics, interrelated and interacting.

Clearly the book is intended for undergraduate students, and it could very well have been a transcription of lecture notes: several times the author uses a purely oral style, like on p. 36: "Suffering is unavoidable here... so please do not start complaining if you are not done in two or three hours..." Sometimes the author uses all-caps words and multiple exclamation marks for emphasis, like on p. 67: "DO NOT stop here! Always look for generalizations and variants! ALWAYS!! Yes, I am shouting... WORK IT OUT!!!" Due to the author's natural charisma, the book is pleasant to read. Furthermore, the fact of wandering through so many areas of mathematics makes one feel confident and opens new horizons.

These are the positive aspects of the method used. But one also could object that there are no solutions given for the many exercises, so one is entirely left on his/her own. Also it is not always clear for what reason the author examines specific topics. A typical example is the issue of calculating the dimension of unit ball projections needed to contain a unit square: what makes this an important problem in the first place?

As already mentioned, it is quite a thrill to discover that just by slightly changing a trivial result one gets hard problems and conjectures. Many statements in the book are Goldbach-like: trivially easy to formulate, and yet unsolvable, or, at least, unsolved up to date. But are they important because they carry the name of some famous mathematician?

Maybe the main interest of the book is to give a sense of unity of mathematics. Its title is "A View from the Top" and indeed one has the impression of being on the top of some skyscraper and looking at the various neighborhoods of a town (neighborhoods called calculus, probabilities, number theory, algebra, Fourier transform, etc.), watching people live their ways in every one of them, and discovering connections and similarities between them.

[14] Transl. Ed. Keeley & Ph. Sherrard, Princeton University Press, 2009.

I recommend this book to people who are curious about discovering things that are usually not taught, nontrivial interconnections and unexpected Goldbach-like conjectures. Time and energy will be needed to, at least, give a try to the many exercises. And the reader will have to try hard not to get frustrated with being unable to solve some/many of them.

The book finishes with a Knuthian exhortation which is so nice and so universal, that I can't avoid giving it here: "Can you anticipate further developments? Can you formulate key questions that could lead to further progress? Are you willing to tirelessly search the research journals and the internet to find out what the concepts you have been introduced are connected to? This book can only be called a success if it causes you to do all these things and more. Good luck!"

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Review[15] of
Geometric Folding Algorithms
Authors: E.D. Demaine and J. O'Rourke
Cambridge Univ. Press, 2007
441 pages, Softcover, $120.00

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1 Introduction

Geometric Folding Algorithms presents a comprehensive discussion of folding (and unfolding) algorithms in Euclidean space. The book is divided into three parts: Linkages, Paper, and Polyhedra. The Introduction (Chapter 0) gives a sample of the problems that will be considered later in the book–some solved, some open–and entices the reader to continue reading.

The target audience for this book is a graduate or undergraduate student in computer science or mathematics. However, most of the book is accessible to a mathematically-sophisticated person with a basic knowledge of geometry and big O notation. We believe that people with even less mathematical background can skim through the book and glean something from the experience.

In addition, the book is so visually appealing that it can be used as a coffee table conversation piece. That use, however, does not do the book justice. For a researcher interested in this area, this book provides a thorough coverage of the current knowledge. Open problems are presented, and waiting to be solved[16].

2 Summary

Part I: Linkages. (Chapters 1-9) This section focuses on one-dimensional linkages. A *linkage* is a collection of fixed length segments joined at endpoints to other segments. Linkage problems investigate both the flexibility of linkages and rigidity or immobility of linkages. For example, a flexibility question could ask if it is possible to turn a polygon inside out with continuous motions (where we allow the polygon to self intersect of course). Contrarily, the linkage referred to in the book as *knitting needles* is an example of a locked chain, where no continuous motion exists that can straighten the linkage. The complexity bounds of both the problems and solutions are presented.

Among the many specific topics included in this part of the book is the history and solution to Kempe's Universality Theorem (KUT). In the 1970's, Thurston colloquialized KUT as "there is a linkage that signs your name." As with all topics covered, the discussion of KUT is complete. A historical significance is discussed, definitions and intuition given, as well as examples along the way, including a discussion of the Peaucellier Link, the first linkage capable of a straight line motion.

One interesting result comes in Section 6.4: every chain (open or closed) and tree of n vertices can be straightened or convexified, without self intersection, in $\mathbb{R}^k$, for $k \geq 4$. Furthermore, this

[16]Authors not: Some have been solved (3 out of around 60). We keep a list of *Closed Open Problems* at `http://www.gfalop.org`.

can be done with a polynomial number of moves and computed in polynomial time. Rather than giving the full proof of this theorem, the authors provide the intuition behind the proof.

In Section 6.6, there is a proof by Connelly, Demain, and Rote that there are no locking chains in 2D brings together many of the topics previously discussed in the book, including: expansiveness, rigidity theory, tensegrity, and flexibility. This is just one of the many examples of how this book is well-integrated. Part I concludes with an application to protein folding.

Part II: Paper (Chapters 10-20) Beginning with the invention of the paper mill, Part II reviews the history of problems that can be imagined by folding paper. This part of the book deals with two main problems:

1. Foldability: deciding if a crease pattern can be folded into origami.

2. Design: deciding if a foldable crease pattern exists that achieves a desired property.

The authors define and discuss mountain and valley folds, simple crease patterns, and flat foldings. In particular, the map folding problem is addressed: given a map that is an $n \times m$ grid of squares with nonboundary edges assigned a mountain or valley fold, decide if the map can be folded flat using only specified folds. For $n = 1$ or $m = 1$ the problem is linear in the number of creases. For $n \geq 2$ the complexity of the problem is unknown.

Chapters 16 and 17 discuss the more "popular" forms of origami and provide the history of the formalization of folding. Folding paper into polygons and polyhedra is first considered, followed by a description of Robert Lang's work on the tree method for creating an arbitrary origami base. Lang's large body of work is condensed very well into a clear high level description.

Chapter 18 discusses designs that can be created from one complete straight cut of a folded paper. Two methods are presented, and a discussion of the two methods reveals that there is a common downside: both methods create extra creases. This discussion leads to the (implicitly stated) open problem: what is the minimum number of creases needed to create a given design? At the very least, can we find a reasonable upper bound for the number of creases needed?

Next, the book looks at constructable numbers. Using only a straight edge and a compass, no construction can trisect an angle. Folding paper, however, can be used to trisect angles. Thus, folding is a more powerful tool than a straight edge and compass. In fact, all quartic equations can be solved by origami; whereas, only quadratic equations can be solved using a straight edge and compass.

Chapter 20 concludes Part II with generalizations of the topics already covered. For example, this chapter mentions David Huffman's work with curved folds (all folds previously discussed were straight). This chapter includes many ideas, but lacks concrete material as this area is largely unexplored.

Part III: Polyhedra (Chapters 21-25) An unfolding of a polyhedra is a set of cuts along the faces and/or the edges that allow it to be unfolded onto a plane. Although all simplicial polyhedra have a vertex unfolding, the existence of a vertex unfolding for general polyhedra is still an open problem.

An example of a non-convex simplicial polyhedra that does not have an edge unfolding is provided. It is unknown if every convex polyhedra has an edge unfolding. Evidence supporting and refuting the existence of such an unfolding is presented in Chapter 22. When cuts along faces are permitted, source unfoldings and star unfoldings provide two positive results. The idea behind these unfoldings are geodesics and shortest paths on polyhedra. Two shortest path algorithms are discussed.

The opposite problem is also addressed: can we start with a (flat) polygon and fold it to a polyhedra? Although every convex polygon folds to an uncountably infinite number of incongruent polyhedra, there exist polygons that cannot fold to any polyhedra. Other topics covered in Part III include: Steinitz's theorem, Cauchy's rigidity theorem, and Alexandrov's generalization of Cauchy's rigidity theorem.

Chapter 26: Higher Dimensions This chapter mainly describes the open problems that are high dimensional generalizations of the topics covered throughout the book.

3 Opinion

Geometric Folding Algorithms provides a thorough coverage of computational origami. The book includes history, definitions, examples, and references. We cannot think of a major related paper or topic that is not considered in some way. Since the text is so complete, it is difficult to identify weaknesses.

However, we have found two weaknesses, which the authors admit in the book itself. First, the book does not include exercises, which can make it difficult for use as a class text. Nonetheless, both authors have successfully used this book in their own classrooms. Some assignments can come from filling in the details of proof sketches, while others could be small implementation projects. As with any geometric code, there are many engineering decisions and degenerate cases that need to be considered. A full implementation of an algorithm presented in the book could constitute a significant project. A more detailed treatment of almost any section could be presented by the students. Second, the book mentions applications, but intentionally does not focus on them. This is not a drawback as much as it is a warning to those who might be looking for applications (for example, in manufacturing).

Now, we wish to highlight the strengths of this book. We found so many strengths that we cannot include all of them in this review.

- The book is well-structured and clearly articulated. Often, the discussion of each new topic begins with a brief intuition, then provides more details.

- When convenient, tables summarize the results. For example, Table 2.1 gives the complexity results for six problems detailed in Chapter 2.

- Throughout the text, the authors present open problems. The book is dense in this area. Nine open problems are explicitly stated in Chapter 22 alone. There are also many implicitly stated open problems. For example, on page 210, a result is established, but the authors comment that a more elegant proof is "within reach."

- Each topic provides ample references. Whether or not details are given in the book, references with deeper coverage of the topic are provided. The authors are clearly selective on what they incorporate, but provide a comprehensive survey of all relevant material. Where details are missing, they refer the reader to the book or paper in which they can be found.

- The authors include interesting tidbits of information. For example, the problem of convexifying a 2D closed chain was originally posed as a homework assignment for an Algebraic Topology class.
- The book is visually appealing, with pictures carefully designed.
- The historical perspective provided with many of the new topics introduced, includes both right and wrong discoveries.
- Since the study of origami spans a large timeframe, and occasionally finds simultaneous solutions, the notations are not aways standardized; however, the authors gracefully handle the notational inconsistencies in the literature.

In summary, *Geometric Folding Algorithms* by Demaine and O'Rourke is a well written and comprehensive text on folding and unfolding polygons and polyhedra. This book is accessible to a novice, but can be used as a reference for the expert. If you were interested enough to read this review, we highly recommend this book to you.

Review[17] of
Geometric Algebra: An Algebraic System for Computer Games and Animation
Author: J. Vince
Springer-Verlag, 2009
183 pages, Hardcover, $70.00

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1 Introduction

Geometric Algebra: An Algebraic System for Computer Games and Animation by John Vince documents the struggle that the author faced with grasping the concepts of geometric algebra (GA) as he wrote his first book on the subject, *Geometric Algebra for Computer Science* (Springer, 2008). Thus, this book is as much a story about geometric algebra as it is a testament to the author's newfound understanding of GA. Now, he revisits the topic with a new perspective by illustrating algebraic structures with colors and tables

Geometric algebra is a framework where geometric objects (lines, areas, volumes, etc.) are treated as vectors. In doing so, we are able to generalize familiar ideas of scalar and cross products of vectors to the inner and outer products of geometric objects. The central concept behind GA is the geometric product, the sum of the inner and outer products. John Vince was astonished that this powerful tool lay latent for many years. As a result, historical contexts are given for most chapters, enlightening the reader to the elegance of GA. Before getting into the summary of the text, we will highlight this historical context. In the 1840s, William R. Hamilton discovered quaternions, which lead naturally to the use of vectors. Thus, mathematicians adopted scalar and vector products. Before this discovery, Hermann Grassmann discovered inner and outer products, an alternative to scalar and vector products. However, his ideas were not adopted, thus hiding GA from the greater mathematical community. The following quote (from Chapter Eight) exemplifies how Grassmann's ideas were largely ignored:

> But what of Grassmann? He had been trying to tell the world that his algebra of extensions was the answer, and although his voice was heard, this message was not understood. This in itself should be a lesson to all mathematicians: keep it simple!

It was not until recently that Grassmann's ideas are finally appreciated. This book aims at demystifying Grassmann's ideas.

2 Summary

The introduction prepares the reader for some of the non-intuitive ideas that can arise in Geometric Algebra, such as multiplying two geometric objects. Then, chapters one through four provide the groundwork for GA. The first chapter reviews the basic real, complex and quaternion products.

The notion of inner and outer set and a novel colored tabular representation of the products are described. The simple derivations in the chapter (as well as throughout the book) are complete and easy to follow.

Chapter two begins by reviewing and illustrating the scalar (dot) and vector (cross) products. As in the last chapter, the tabular form is used. The outer product is defined and its geometric interpretation as a signed oriented volume is discussed. Both 2D and 3D examples are provided with ample illustrations.

Then, the geometric product is introduced. This is a product of two vectors a and b, using the inner and outer product; that is, $ab = a \cdot b + a \wedge b$. A few simple examples in 2D and 3D are provided, to familiarize the reader with the algebra of the geometric product. As in the previous chapters, colored tables are used to illustrate the patterns that emerge in computing these products.

Next, the author uses simple algebraic manipulation to redefine the inner and outer product in terms of the geometric product. We see that for vectors $a_1, \ldots, a_n$, the outer product $\wedge_{i=1}^{n} a_i$ can be written as $n!^{-1}$ times a sum of geometric products.

The author then defines the outer product of k vectors as a k*-blade*, where k is the *grade* of the k-blade. We see that the number of basis k-blades in dimension n is $\binom{n}{k}$. Finally, we see three example of how the geometric product behaves in 2D for orthogonal, parallel and linearly independent vectors.

Chapter Five introduces a new algebraic structure, the *multivector*, which is a linear combination of elements of different grades. The geometric product with multivectors is explored. The associative and distributive properties of multivectors are mentioned.

Given a geometric product of vectors $A = abcd$ the *reversion* of A, notated as $A^\dagger$ or $\tilde{A}$ is $dcba$. The inverse of a multivector A is a multivector A^{-1} such that $AA^{-1} = 1$. The author derives the inverse of A as $A^\dagger/(A^\dagger A)$. Examples are provided for both reversion and inverse, stressing the non-commutativity of the geometric product.

Next, the similarities between the outer product and imaginary numbers are explored. For example, taking the geometric product in 2D of a vector v by the unit bivector e_{12} on the right, i.e. ve_{12}, rotates v $\pi/2$ degrees counter clockwise. Chapter Five concludes with a description of duality in geometric algebra with specific examples of duality between vectors and bivectors in 3D.

Chapter Six and Seven look at the details of the inner and outer geometric product in 2D and in 3D. Details are provided for the product of all basis blades. These two chapters contain examples that can clarify uncertainties that the reader may have had about the products.

Chapter 8 introduces reflections of vectors, bivectors and trivectors about a line and a mirror. For a k-blade B, the reflection about a line m is $B' = mBm$ and reflection in a mirror with normal defined by m we have $B' = -mBm$ for vectors and trivectors and $B' = mBm$ for bivectors, scalars are invariant. The author then describes how rotations can be achieved from two reflections, called a *rotor*, and relates the rotor to a matrix representation of a rotation that is used for quaternions. Next the author discusses building a rotor R from two vectors and applying R to form a rotation and provides an example of the construction. He builds up to describing interpolation between rotors by first describing a simple interpolation between scalars. Then he derives a spherical-linear interpolation between two vectors. He mentions that the same derivation can be applied to rotors and quaternions. The chapter concludes with a simple example of interpolating between two rotors.

Finally, Chapter Nine applies GA to 2D and 3D geometric problems. Since Chapters One through Eight were very computational, this chapter helps to give the reader a perspective on the subject. It shows not only what these products are, but how to use them.

3 Opinion

When reading a technical book, a reader can become lost in the details, often missing a crucial step in the process being described. This is not the case with *Geometric Algebra: An Algebraic System for Computer Games and Animation*. In this book, every line is clearly derived from the previous line. Thus, an undergraduate student or a working professional with a very little formal mathematical training can use this book for self-study. The book is easy to read, as it follows a very standard format: each chapter begins with a short introduction, and ends with a succinct summary. In between, algebraic manipulations are the heart of the text.

John Vince's goal was to lay out geometric algebra so that it can be employed by anyone working in computer graphics. We believe that the author has far surpassed this goal. The text is so clear that the operations described in Chapters Six and Seven were implemented with ease, even though no discussion of implementation was made in the text. The summaries at the end of each chapter were succinct, and meaningful.

One of the highlights of the text is the great use of tables and colors to emphasize the ideas and make them easier to remember. In fact, the use of color to emphasize the ideas was very successful, although there may be an issue with some of the images for a color vision deficient person, especially when red basis vectors are drawn on a green area.

One reason that this book is so clear is that it is very focused. The primitives are given before applications are discussed. For specific topics discussed, the mathematics is explained and then the geometric meaning is explained. Furthermore, the ideas of one chapter are built upon the ideas of the previous chapter naturally.

In summary, *Geometric Algebra* by John Vince is an excellent way for a student or a professional to learn about geometric algebra. The text is clear, and no steps are skipped. After reading this book, the reader should be able to use the primitives of geometric algebra. From there, many geometric operations and constructions can be created without much effort.

Review[18] of
Dude, Can You Count?
Stories, Challenges, and Adventures in Mathematics
by Christian Constanda
Published by Springer, 2009
226 pages, Hardback, $27.50
Review by
William Gasarch gasarch@cs.umd.edu

1 Introduction

This book is a vigorous criticism of many things in our society. Some of them are mathematical, some of them are not; however, mathematics permeates the entire book.

I am sure that some readers will say things like

When he complains about those damn journalists who don't care about our national security I think RIGHT ON!, but why is he so down on Calculus Reform?

That is, people will agree with parts of it and disagree with other parts of it. He brings up interesting points, both within mathematics and not. I disagree with him on some of those points. More often I think *the issue is more complicated then he indicates.* However, the point of the book is to get you to think about things. At this it succeeds. It often does so in satirical ways which may seem harsh; however, he is trying to shake us out of our complacency. I hope he succeeds.

The book was written to get people to think and talk about the issues raised. Hence what's the best way to review it is to talk to the author. After the first section of this review *Dude- Whats in this Book?* I have three sections that ask for (and get) the author's response: *Dude- Please Expand on That*, *Dude- Do you Really Think That?*, and *Dude- That comment was so Jiggy that I want to, like, add to it.*

2 Dude- What's in this Book?

The book is a conversation between a space alien and someone (the author?) about what the alien sees on earth. Each chapter begins with several math problems and then goes onto other things. The math problems are interesting and mostly elementary. I suspect that 1/3 of my readers will have seen about 1/2 of them. However, as he writes in the preface, he did not insert them for their novelty, but because they illustrate important mathematical points and show how mathematics should be handled correctly.

Each chapter is on a different topic. Each chapter is individually very interesting. I am not quite sure what it adds up to in the end. Possibly that people are illogical and hence if math was better taught this would be a better place. However, we've all known logical people who are awful human beings, and illogical people who are fine human beings, so that does not quite work.

Who should read this book? The math is not so hard- a high school student could read it. It provokes thought on mathematics *and* the real world which has got to be a good thing. It raises some uncomfortable issues which again has to be a good thing. I do have one caveat, not on whether to buy the book— I recommend that you buy it— but on how to read it: do not read it all at once (as I had to, to get this review done) since it's a bit much to take in all at one time.

3 Dude- Please Expand on that

1. On Page 7 you seem to say that the MICQ measures *a continuum of variables collating their past and present values into a comprehensive picture of one's capacity for logical thinking, common sense, imagination, culture, and overall personality.* This is a lot of unconnected qualities to put into one number. How would an Assassin (low values) who is creative, logical, imaginative, cultured, and a very nice guy do on the MICQ?

   [Author: *An assassin would have a zero MICQ, however intelligent he may be. The MICQ is defined (top of page 5) as the absolute minimum of the curve. See the second and third graphs and accompanying comments on page 5. Obviously, a human cannot blend so many disparate qualities into a single number, but JJ is not human and his kind can do it. This is a piece of cake for folk who can levitate and walk across a body of water (page 233).*]

2. On Page 93 you explain Calculus Reform as having the students work in groups and using computers and some formal manipulation (without teacher involvement) and (I assume this is your objection) no rigor. This raises the question that I ask you non-rhetorically: what level of rigor should non-math majors taking calculus have? Epsilon-Delta proofs seem like too much, but just pressing FIND AREA on a computer seems like too little.

   [Author: *Personally, I would be happy if these students got an intuitive representation of the concept of limit and understood the mathematical procedure for computing the area. They shouldn't be burdened with the epsilon-delta proof of it. Math majors are a different proposition altogether. My main objection to the so-called Reformed Calculus is that it forces students to try to discover for themselves a concept that has not been defined to them. Hence the analogy with the dark room in the book.*]

3. Page 278. You suggest that mathematics has been responsible for much of today's technological progress and inventions. I agree! However, could this have all been done with non-rigorous applied mathematics? How important was making calculus rigorous to its application? I ask this non-rhetorically.

   [Author: *Mathematics is responsible for a lot more than that—for example, it teaches people to reason and behave logically, and elevates human thinking and perception of the universe to higher and higher planes. Basic technology could have advanced empirically on its own, of course, but with a lot more blunders on the way and not as fast as it has under the guidance of mathematics. "Non-rigorous applied mathematics" is a contradiction in terms because all mathematics, whether pure or applied, is by definition rigorous.*]

4 Dude- Do you really think that?

1. On Page 28. You ask the standard Socks Problem: Which I abbreviate here: *There are 6 blue socks, 8 white socks, and 10 black socks in a drawer. How many do you need to take out of the*

drawer in order to get a pair. I answered 4. So did some very talented math people I asked. You say it's 2 since you did not insist that a pair of socks both have the same color. This is a nice trick question. However, you use students performance on this problem (80% got the answer 4) to make the point that students should read questions more carefully. This is not fair since it is a trick question.

[Author: *The explanations given there and on page 29 simply point out that students must learn to read their homework/test questions well. There is no such thing as a trick question in mathematics.*]

2. On Page 44 you complain about slang becoming part of speech. But one of the great things about English is its ability to adapt. New words come into the language- what is wrong with that? Below are some new words or phrases that have entered our language in the last 20 years. Some of them may have started out as slang. Attention Deficit Disorder, Birthers, Blog, Civil Union, Cougar, Disemvowled, Friends with benefits, Funemployment, Gadar, Googled, Helicopter parents, Infotainment, Lifosuction, McMansion, Nascar dad, Obamacare, Person of interest, Playdate, Podcast, Ringtone, Snail mail, Soccer mom, Staycation, Swift boated, Text (as a verb), Truthiness, Tweet, Viral Video, Wiki. (My favorite: Disemvowled, which is to take a text and remove all of the vowels. Its a way to punish someone who writes a nasty comment on a blog without actually censoring him.)

   [Author: *I do nothing of the sort. Lin's first answer on that page acknowledges this as a positive feature of the language. Immediately below, Gan says that allowing new words into English is just fine if they "bring richness of meaning or invigorate aesthetic expression". The words you mention are fully acceptable when they are used with their proper grammatical function. My complaint is about the misuse of existing words, such as "like". By the way, did you notice that I introduced a new word myself? It is "semidoct", first used on page 22 and explained on page 25. You will not find this word in any English dictionary. Yet!*]

   [Reviewer: My bad. I should have, like, read that part of your book more carefully dude. And semidoct is just so awesome a word!]

3. Page 109. A mathematical mistake! (the only one I found). You claim that *for a long time it was thought that Fermat's so-called last theorem might be one of them.* By this you meant might be a statement that is independent of Peano Arithmetic or ZFC or what have you. Actually, this is impossible. If FLT is independent then there is no proof of FLT or its negation. However, if the negation is true there is an easy proof of it- the actual numbers (x, y, z, n). Hence if FLT is independent then it is true. The same type of reasoning works for the Goldbach conjecture.

   [Author: *Please note that I only reported what some people thought might be the case. I did not say that I agreed with their belief. If I say that someone thought he had seen a flying saucer, it doesn't mean that I accept the existence of flying saucers, JJ's vehicle notwithstanding!*]

   [Reviewer: Did people really think that? The proof that it cannot be the case is so easy that this surprises me. Do you have a source?]

   [Author: *See the article "Is Mathematics True?" by Roger Luther, published in Mathematics Review, vol. 1, no. 5, 1991, pp. 29–31. This was before Fermat's Last Theorem was proved. The article also mentions the Goldbach Conjecture.*]

4. Page 112. You point out the immorality, especially for medical drugs, of advertisements. However, while I agree that ads for drugs are sometimes misleading you seem to trust doctors too much. Doctors often say *in my experience this brand works* which may be correct, but not scientific. Worse still, some doctors get free samples form drug companies and recommend them without real proof that they work. So, while I agree that drug companies sometimes do bad things, doctors do too. There is enough unethical behavior to go around.

   [Author: *Certainly. But there are only so many pages in a book, so I had to make a selection. The doctor in the short dialog is showing his annoyance at the patient. I also mention the freebies doctors get from the drug manufacturers—see the first paragraph immediately below the reported dialog on page 113.*]

5. Page 122. You think that cameras at traffic lights are just fine and that civil liberty types who object are wrong. The issue is far more complex than you indicate: (1) cameras that start out just for traffic violations may be used to catch people breaking other laws. What is wrong with that? What if a society has very stupid laws? Or what if completely innocent people get harassed. This is not hypothetical.

   [Author: *This is page 127, not 122. To be brutally frank (and somewhat cynical), a democratic society with very stupid laws deserves them because a majority of its members voted for the politicians who made those laws or refuse to change them. (In a totalitarian regime, the citizens have no say in anything, including traffic cameras or electronic bugs in their homes.) Road cameras are already being used to identify criminals committing crime or running away from a crime scene, and this kind of evidence is perfectly acceptable in court. I have also pointed out the controversy surrounding these cameras, on the lower part of that page. I should say (from direct knowledge—I lived there for almost thirty years—that speed cameras work very well in the UK and nobody is complaining about them. As a consequence, they have a far lower road accident rate than here.*]

   [Reviewer: Do we truly have a democratic society? Between voter fraud and corruption this is not clear.]

   [Author: *A good question. But I won't comment on anything political. Not in this context.*]

   (2) If there are cameras at lights then the mentality becomes *if there is no camera then it is okay to speed or go through a red light.* More generally, balancing public safety against legitimate privacy concerns is difficult.

   [Author: *Yes, it is, and I don't claim to have a solution. But I disagree with what you say: only irresponsible drivers, to put it mildly, would reason the way you suggested, the kind of drivers who would not be deterred from wrong-doing even by the presence of cameras.*]

6. Page 186. You blast journalists who would endanger our national security by printing classified information. However, it also works the other way around: Governments claim things are classified that are merely embarrassing. There is enough unethical behavior here to spread around.

   [Author: *Absolutely! But I was told to keep out of religion and politics, so I could not mention any of the government's shenanigans. I was barely allowed to use GAWD and Gate Keeper in the jokes—it was felt that these substitutes would not give offense.*]

7. Page 203. You suggest that rather than have people tried before an uninformed irrational jury they should be tried in front of an informed rational judge. This may give the state too much power. Juries were originally (and still can be) a check on government power.

   [Author: *Judges can always set aside a jury verdict if they find it totally unreasonable, but they are reluctant to do so because they don't want to be appealed. I suggested in the book that judges should be given the power to ask questions. Then defense attorneys would have their wings clipped a bit and not be able to pull the wool over the inexperienced jurors' eyes. This might be a reasonable middle-of-the-road solution that would make jury verdicts more informed. But it will never happen.*]

5 Dude, that comment was so Jiggy that I want to, like, add to it

1. Page 137. You discuss America's inability to go to the metric system. No argument here; however, (1) Here is a funny video about when the British went to decimal currency: It is a song by Tom Lehrer

   `https://www.youtube.com/watch?v=ntW9NDRHXnc`

   [Author: *Droll. However, the Brits changed to decimal currency overnight and all got used to it right away, so it can be done successfully. Where there is a will, there is a way.*]

   (2) Here is a funny article in *The Onion* (a satirical newspaper) about how inner city kids learn the metric system pretty well (e.g., 9mm gun, kilos of cocaine,...).

   `https://www.theonion.com/metric-system-thriving-in-nations-inner-cities-1819565900`

   [Author: *I made a passing mention of this in the second motto on page 133. I could not have elaborated the issue in the chapter because it would have made the book politically incorrect and therefore unpublishable.*]

2. Page 151. Credit Card Debt. First off, I agree that people often buy stuff they cannot afford and don't really need. See this great Saturday Night live sketch:

   `https://www.youtube.com/watch?v=R3ZJKN_5M44`

   However, there are people who use credit cards responsibly so I would not call credit cards an invention of the devil.

   [Author: *This line is part of a joke, not a statement of fact or even a personal opinion. After all, I use credit cards myself, but pay the balance by direct debit every month. I may, nevertheless, be sent straight to hell when I die!*]

3. Page 159. Modern Art. It is even worse than you wrote. See

   `http://stubbornwriter.com/2010/05/13/09/29/37/entertainment/movies/selling-feces-as-art-sas`
   `352`

   [Author: *Oh, I agree!! But again, the publisher told me to tone down my criticism. This is one of two chapters that I had to rewrite completely because the original was scathing.*]

4. Page 165. You discuss if science can help art. I agree that it can. The comedian Jerry Seinfeld said: *I was great at Geometry. If I wanted to train someone as a comedian, I would make*

them do lots of proofs. That's what comedy is: a kind of bogus proof. You set up a fallacious premise and then prove it with rigorous logic. It just makes people laugh. You'll find that most of my stuff is based on that system… You must think rationally on a completely absurd plane.

[Author: *I do exactly this in the book myself, with the "proofs" of some outrageous statements.*]

5. (Related to the last item) Also if interest: A proof that the Halting problem is undecidable as a Dr. Seuss poem:

   `http://www.cs.umd.edu/~dml/seuss-halt.pdf`

   [Author: *A very witty poem.*]

Usually in a review I do all of the talking. Since in this one the author is responding to my questions I want to thank him and give him the last word:

Author's General Remark. *As explained in the preface, this book is a tongue-in-cheek comment on the state of education and other aspects of our society that, in my personal opinion, leave a lot to be desired. It is not meant, nor does it claim, to cover all the angles or offer solutions to the issues. Any expectation to the contrary is therefore unjustified. The book's goal has been pursued through mathematical fact or analogy. Its mathematical features are also intended to have a pedagogical value, of use principally to high-school students and freshman and sophomore undergraduates. It is in this spirit that the book should be read and interpreted.*

I would also like to thank the reviewer for his fair comments and relevant questions, and for adhering to that sound Roman precept, audiatur et altera pars.