The Book Review Column[1]
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Erata: In the last issue, Volume 44 No 4, The review of *Computability and Complexity Theory* by Steven Homer and Alan L. Selman, reviewed by Jeffrey Shallit, was accidentally cut short. The full review is available on the electronic version at www.cs.umd.edu/~gasarch/bookrev/44-4.pdf

In this column we review the following books.

1. **In Pursuit of the Unknown: 17 Equations That Changed the World** by Ian Stewart. Review by Omar Shehab.
2. **Unauthorized Access: The Crisis in Online Privacy and Security** by Robert H. Sloan and Richard Warner. Review by Harry Lewis
3. **Boolean Functions - Theory, Algorithms, and Applications**by Yves Crama and Peter L. Hammer. Review by Haris Aziz.
4. **Additive Combinatorics** by Terence Tao and Van H. Vu. Review by Raghunath Tewari.
5. **Discrete and Computational Geometry** by by Satyan L. Devadoss and Joseph O'Rourke. Review by B. Fasy and D. Millman.
6. **Iterative Methods in Combinatorial Optimization** by by Lap Chi Lau, R Ravi and Mohit Singh. Review by Yang D. Li.
7. **Perspectives on Projective Geometry** by Jürgen Richter-Gebert. Review by S. C. Coutinho, 2012.
8. **Who's #1?: The Science of Ranking and Rating** by Amy N. Langville and Carl D. Meyer. Review by Nicholas Mattei.
9. **Boosting : Foundations and Algorithms** by Robert E. Schapire and Yoav Freund. Review by Shiva Kintali.

BOOKS I NEED REVIEWED FOR SIGACT NEWS COLUMN
Algorithms

1. *Greedy Approximation* by Temlyakov
2. *Algorithmics of matching under preferences* By Manlove.
3. *Data clustering: Algorithms and Applications* Edited by Aggarawal and Reddy.
4. *Modern Computer Algebra* by Gathen and Gerhard (Third edition-2013).
5. *The LLL Algorithm.* Edited by Nguyen and Vallee.

Misc

1. *Proof Analysis: A Contribution to Hilbert's Last Problem* by Negri and Von Plato.
2. *Introduction to reversible computing* by Perumalla.
3. *The Block Cipher Companion* by Knudsen and Robshaw.
4. *Distributed Computing through combinatorial topology* by Herlihy, Kozlov, Rajsbaum.
5. *Towers of Hanoi— Myths and Maths* by Hinz, Klavzar, Milutinovic, Petr.
6. *A Mathematical Orchard: Problems and Solutions* by Krusemeyer, Gilbert, Larson.
7. *Mathematics Galore! The first five years of the St. Marks Institue of Mathematics* by Tanton.
8. *Six sources of collapse: A mathematician's perspective on how things can fall apart in the blink of an eye* by Hadlock.
9. *Selected Papers on Computer Languages* by Donald Knuth.
10. *Handbook of finite fields* by Mullen and Panario.

Review of[2] of
In Pursuit of the Unknown: 17 Equations That Changed the World
by Ian Stewart
Basic Books, 2012
330 pages, Hardcover

Review by
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1 Introduction

This is the latest spell from the 'Honorary Wizard of the Unseen University'! Let me warn you that it will take a while for you to come out of the three hundred and thirty pages long spell. Of course, you will not repent! I bet whoever reading this review has written quite a number of equations in some part of her life. It must have started at the school unless you are a Terry Tao or Sheldon Cooper. In school days, equations, some times, may have changed our own lives probably because of writing them wrong in the exams. But there have been people in the history who wrote correct equations and changed the world by writing it for the first time! Ian Stewart picked seventeen of them which he thinks and I agree that influenced us significantly. I am not sure why he picked exactly seventeen equations but I am happy as long as it is a prime number!

2 Summary

The book dedicates separate chapter for each equation. I would like to warn the readers about a caveat here. Although the title may seem to be focusing on 'equation', the focus is and should be actually on 'changing the world'. It really takes more than just an equation to change the world. It takes you and me, the people who appreciates it along with the band of geniuses who worked it out throughout the course of history. Moreover the people and the events needed to be positioned perfectly along the time line to have an impact. So, when Ian Stewart makes the case for a particular equation the story goes far beyond the symbols. It starts with the historical background about how the equation was first anticipated. Then he goes on telling how people generalized the ideas and formalized the results. Finally he describes how the equation gives birth to new branches of science and how it keeps influencing our lives till today.

Being a master story teller, Ian Stewart knows how to entertain you even with Greek symbols! Here, I pick seven equations from the book and share the fun I had while I was going through the chapters.

2.1 $a^2 + b^2 = c^2$

You got it right! It is something about Pythagoras. But did you know the Babylonians knew about it even thousand years before him? Let's see what it says. According to a song in the 1958 movie Merry Andrew,

The square on the hypotenuse
of a right triangle
is equal to
the sum of the squares
on the two adjacent sides.

As the chapter hints, the idea was triggered by the measurement of farm lands and took a while to be expressed into the equation form. From its inception the equation found its way into map making, navigation and surveying. Most importantly it gave birth to trigonometry which means 'triangle measurement'. To justify this the author gives a very brief but convincing introduction of how trigonometric identities could be related to the equation. Then he goes on describing how coordinate systems were devised with the help of the equation. At the end of the chapter the author describes arguably the most important contribution of the equation. This time not by using its results but identifying the cases where the geometric inference of the equation doesn't hold true. It is Riemannian geometry! Think about a BIG right triangle drawn on the surface of the northern Hemisphere of our mother earth. You can already see that due the curvature of the earth you are going to get different relations among the squares drawn on different sides of the triangle. This discrepancy was first found in Euclid's proof of the theorem where he assumed that the geometric surfaces are always flat. It was not true. When mathematicians tried to prove the theorem without using Euclid's assumption it gave birth to Riemannian geometry which is now-a-days the de facto language for studying cosmology. Isn't it entertaining to read how the proof you learnt at school has created a whole branch of mathematics which explains the universe?

2.2 $\log\ xy = \log\ x + \log\ y$

If you have someone in the family who is going to middle school show the first page of this chapter to her. I bet she will be shouting, "That's LOG! I know how to do it!". Now tell her that when the English mathematician Henry Briggs went to meet John Napier, who invented logarithm, he was already so impressed by reading the book that he kept shaking Napier's hand for fifteen minutes! Like today, multiplication was one of the most difficult and time consuming arithmetic operations back in those days. That's why they have cash registers at McDonalds. You (along with your young friend) must be very excited to know what was the initial flaw in the intuition for logarithm and how it was fixed by Briggs. Although Napier devoted his time to finding spiritual insights from this own findings, other people started building large logarithmic tables so that all could reuse the already worked out results. The chapter very nicely illustrates how it saved huge amounts of time for astronomers, surveyors, and navigators— enabling them to focus more on astronomy, surveying and navigation. It also explains how the ideas of logarithm helped designing slide rule, the most important arsenal of an engineer before calculators were invented. Then the author briefly explains how some of the natural processes are inherently logarithmic. I was really startled to know by the end of the chapter that sensation perceived by human is proportional to the logarithm of the stimulus!

2.3 $\frac{df}{dt} = \lim_{h \to 0} \frac{f(t+h)-f(t)}{h}$

The chapter starts with a personal touch about the time of England when Newton was going to the Trinity College in Cambridge. The initial ideas of Calculus were hanging around for about a century. The author explains how Galileo's understanding of the motion of falling bodies and Kepler's understanding of that of planets set the bed for Newton's contribution. Then comes the infamous dispute of plagiarism between him and Leibniz. The author very nicely shows how different were the two intuitions behind the same formalism. The discussion is followed by an even more interesting story of the spiritualist George Berkeley. He interpreted calculus as a framework which explains the way the universe is without borrowing ideas from religions which was blasphemous! Leibniz also had a critic who disputed the idea of infinitesimal terms. Even before the foundation was strengthened by enough mathematical rigor, the experimentalists started harvesting the fruits of new framework. Then the chapter very concretely describe how the concepts in mechanics, for example, velocity, speed, momentum, angular momentum etc., were formalized in terms of calculus. It took another century though for the mathematicians to rigorously define the infinitesimal and instantaneous quantities (the field now called nonstandard analysis). But rigorous or not, after people started applying calculus the world was never same again.

2.4 $F = G\frac{m_1 m_2}{d^2}$

The chapter confirms that the story of apple falling on Newton is incorrect. It starts with the discussion about how the ground was prepared for Newton when he started his journey of understanding the universe. One of the giants on whose shoulder he was standing was Kepler. Kepler thought his understanding of planetary motion reflects the existence of a mystic higher level driving force of nature. It answered, at least to some significant level of approximation, how the celestial objects move. The question of what makes them move was not answered by Kepler's laws. That's where Newton's work started from. Using the ideas and observations, hanging around for a while, Newton was able to propose a law about how two body attract each other. This time again, his publication made Robert Hooke feel that his idea was stolen. The treatment of this chapter proves how efforts in various way were going on to solve the problem. As the chapter goes on, Newton's law of gravity opened up a new avenue of successive efforts on related problems including the three body problem tried by Poincaré. Although an approximation, Newton's theory is heavily used in practice till today. The chapter illustrates how orbits are design in space mission and fuel efficient inter-planetary travel routes are calculated using the principle. For example, the interplanetary tubes and their junctions are direct implications of the scientific efforts which was started by Newton centuries ago. It also helped in finding the Lagrange and Trojan points crucial for designing space missions. After landing on the moon, Neil Armstrong said, 'That's one small step for [a] man, one giant leap for mankind'. The first small step itself was a giant leap by Newton when he tried to explain why planets travel without considering the existence of the will of any super natural being.

2.5 $i^2 = -1$

The imaginary number, i, is as real as you are! So, it is a misnomer. As soon as algebra became a favorite way to the mathematicians in representing the real world, imaginary numbers started to appear in different solutions. The chapter gives an intuitive idea of how it could emerge even in arithmetic. The story starts with the highly eventful life of Cardano, who discovered a procedure for solving cubic equations. Some of the cubic equations, like $x^3 - 15x - 4 = 0$, may have solutions containing square roots of negative numbers, it our case $x = \sqrt[3]{2 + \sqrt{-121}} + \sqrt[3]{2 - \sqrt{-121}}$. Back then nobody had ever used the number $\sqrt{-121}$ in real life. In some cases such quantities could be canceled out after simplification but not always. So, the mathematicians were not very happy with that. Since then finding their significance kept a lot number of bright mathematicians busy. The chapter illustrates how Leibniz, Wallis, Wessel , Argand and Gauss worked in different times to build a framework for representation and manipulation of imaginary numbers. It encouraged the mathematicians to apply the already established branch of mathematics, for example calculus, trigonometry etc., for the cases of imaginary and complex numbers. The imaginary number was eventually found to be a foundational element of mathematics as illustrated by the famous equation $e^{i\pi} = -1$. These efforts built the analytical foundation for the revolutions which later took place both in science and engineering starting in eighteenth century. Let me rephrase the first line of this section, the imaginary number, i, is as imaginary as you are!

2.6 $F - E + V = 2$

This is the relation among different properties of polyhedra. F, E and V indicate the numbers of faces, edges and vertices of a polyhedron respectively. The formula is due to the person who once said, 'Cogito ergo sum'. Descartes considered this as a trivial discovery and moved on. After more than a century, Euler established the foundation of this relationship mathematically. The chapter gives an intuitive understanding of how the proof works. The discussion also gives the initial intuition of topology which is discussed eventually. Euler characteristic built the foundation for modern day topological invariants. The chapter then discusses how these invariance play roles in classifying, among other topological objects, the counter-intuitive constructs like Möbius band and Klein bottle. The discussion then leads to the concepts of Knots, a one dimensional embedding in three dimension with incredible potential for explaining broad spectrum real life phenomena. It describes the basic rules of knot dynamics and relates it to the polynomials which remain invariant over knot transformations. One of the early invariants is the Alexander polynomial. Incidentally, the chapter mentions the contribution of my academic grandfather, Ralph Fox, who contributed in proving the strength of the polynomial. The discussion ends with brief examples of how knot theory is used for understanding DNA folding and Interplanetary Superhighway. It all started with the equation Descartes was careless about!

2.7 $\phi(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu^2)}{2\sigma^2}}$

If you want to know what is the ratio between the reading time and number of words of this page for most of the readers, you will find that its log follows the equation in the title. This is called normal distribution. It started with a gambler cum mathematician, Cardano. Later formalized by Pascal and Fermat again on the questions about gambling. Later it finally saw a rigorous treatment by Bernoulli. He formalized a nice way for predicting the outcome of a series of repeated events. It quantitatively explained how chances of winning and loosing evolve in gambling as the game proceeds. But for longer series, often appearing in real life, it was not computable in a reasonable time. We can do this now a days with computers but Bernoulli used to live in sixteenth century. Later De Moivre came up with the definition of the normal distribution as above to approximate the outcome for large number of repeated events. The distribution looks like a bell curve which means the closer an event is to its mean the higher is its probability of being observed. Eventually, people found many other real life phenomena which could be represented by normal distribution. Observation of the distribution in unrelated cases made people think about whether statistical results in, for example, demography are meant to be as they are. Even projects like, Eugenics, had been taken to manipulate the natural evolution of population which was highly unethical. Eventually, discourses are converging more and more everyday about in which cases we should push people who are on the tails of the bell curve towards its mean. The equation of this chapter's interest is exerting far more implications than mere gambling.

3 Opinion

Ian Stewart is not a new name in popular science. This book is just another addition to the popular titles, manifesting his genius, which include *The Mathematics of Life*, *Why Beauty Is Truth: A History of Symmetry* etc. The best parts of the book are the many stories you would wish you had been taught when you first learned these equations. Very few people could imagine that the GPS, they are using everyday, might have anything to do with General Relativity which is mentioned by the press only when there is a news about Stephen Hawking! Less imaginable is about the current financial crisis which is partially caused by an inappropriate use of a great equation. Our lives are full of equations. It is the genius of the scientists and engineers which conceal them with cool solutions. They ensure that we can worry only about lives and not about equations. But who told us that equations are only for worrying about! Aren't they fun too? You must agree if you read the book! You bet!!

Review of[3]
Unauthorized Access: The Crisis in Online Privacy and Security
by Robert H. Sloan and Richard Warner
CRC Press, 2013
398 pages, Paperback $50.32

Review by Harry Lewis (lewis@harvard.edu)

1 Introduction

In early 1968, when Christopher Jencks and David Riesman chose the metaphorical title *The Academic Revolution* for their scholarly, sociological study of American higher education, they could not have known that shortly after the book appeared in print, American universities would be in the throes of what looked like an actual revolution, with massive protests, live gunfire, students killed, and police officers injured. I am reminded of this naming overreach because a reader in 2014 will surely open a book about a "crisis in online privacy" expecting to learn something about the world in which Edward Snowden revealed we are living—the world of ubiquitous telephone surveillance, government iPhone hacking, and laptops intercepted during shipment so spyware can be installed. Readers hoping to learn something about such forms of unauthorized access will not find them discussed in *Unauthorized Access*. Nor should readers think that the title is quoted from the Computer Fraud and Abuse Act, or is some other allusion to the digital-era Fourth Amendment.

Unauthorized Access is instead entirely about a different and also important area of public concern: the behavior the private sector—the commercial aggregation of data for online advertising, the commercial production of software that is vulnerable to malware, etc. As the authors (a computer science professor at the University of Chicago and a law professor at the Illinois Institute of Technology Chicago-Kent College of Law) explain,

> We limit our discussion of privacy and security to the private sector. For security this is not a significant limitation. Everything we say also applies to governmental computers and networks. Privacy is a different matter. Governmental intrusions into privacy raise legal and political issues that don't arise when private businesses encroach on privacy. However, the chorus of concern over government intrusion into our private lives is already large and strong. Moreover, the threat from private business merits consideration on its own. (p. 1)

All true. But the chorus about government intrusions has gotten a lot louder lately, and its volume is likely to make it harder for the authors' voices to be heard on the subject of corporate behavior.

2 Summary

The book is a systematic analysis of private-sector data privacy issues from a game-theoretic standpoint. An early summary lays out the approach.

> We argue that profit-motive-driven mass-market sellers often offer consumers products and services inconsistent with consumers' values. Our proposed solution does not try to change sellers' motives. We focus instead on buyers. Our view is that, for a variety of reasons, consumers tolerate, and sometimes even demand, products and services that are in fact inconsistent with what they value. Our solutions bring buyers' demands in line with their values, with the result that buyers will demand greater security and more privacy. We argue that the profit-maximizing strategy for a mass-market seller is to meet the changed demand and offer products and services consistent with it. (p. 9)

The book proceeds from first principles and the prose is clear and mostly free of jargon. There is no math and almost no computer science. A couple of chapters on computer and network technology are included to make the work self contained but are rather beside the point—computer scientists will skip them because they are too familiar, and readers more interested in law, policy, or economics will skip them because many of the details don't matter much to the general argument. The bulk of the actual work is in developing a theory of "norms"—collective behavioral regularities—that are "value optimal"—generally regarded as equally well justified as any of the available alternatives—and applying these concepts to the multiple tradeoffs in the games that businesses and consumers play with each other (you let me collect data on you, and I will give you more relevant advertising and special deals, but maybe also sell the data I have collected to others without telling you, but if you find out you may defect to a competitor, in which case I will . . . , etc.).

I am not an economist or game theorist, and I don't think the authors are politicians or political scientists. As a result, it is hard for me to be sure whose blind spots are more serious as I try to assess the realism of the authors' proposed solutions to the dilemmas of privacy and security. For example, the authors say,

> With the notable exception of behavioral advertising discussed in Chapter 12, we need to rely on legal regulation to achieve our results. However, effective legal regulation is difficult and expensive, and for this reason, we rely on legal regulation only temporarily, in the initial stages of our solution. Our processes ultimately lead to social norms with which business will voluntarily comply because compliance is profit maximizing. . . . There are some key results about privacy we cannot achieve unless businesses internalize values that constrain their pursuit of profit. This would be a major cultural shift. (p. 9)

Well, yes. It is hard to get laws passed, especially, in this political climate, laws restricting the conduct of business. What sort of laws do the authors have in mind? In Chapter 7 they propose a solution to the problem of vulnerable software, which gets a lot attention:

> Require that developers file and annual, publicly accessible report that details their development and programming practices. The goal is to ensure that enough buyers know whether or not a seller offers best practices software to make offering such software the profit-maximizing strategy. (p. 171)

OK. So if this were all in effect today, Microsoft and Facebook would have to publicly disclose their software engineering practices, so that potential users could scrutinize them and decide to move to other platforms if they thought, say, that "Move quickly and break things" did not represent the best practices of the software engineering industry. There are many reasons why this scenario could not happen today, and indeed should not happen today. Trade secrets. Burden on startups. Damping of innovation. Network effects limiting users' real choices. Users' inability to assess the state of the art. The likelihood that any "Consumer reports" analog would be outgunned by the industry it is trying to keep honest.

But, the authors tell us, the legislative fix is short term—they are talking about the long term. The authors write,

> Creating the norms we need requires reaching agreement on extremely controversial trade-offs against a background of constant change and innovation. That is not likely to happen quickly. It is difficult to predict how long it will take, but it would not be surprising to find the following observations in a twenty-second century history of the early decades of the information era: "In the early decades of the twenty-first century, societies struggled to balance privacy against he benefits created by exponential increases in the power to collect, store, and analyze information. The norms that we take for granted today first emerged only toward the middle of the century. (p. 341)

Now any argument about how market forces will push the information technology industry over a decade, much less over a half-century, requires lots of assumptions and acts of faith. And almost every argument the authors make for how norms will evolve is based on some prediction of seller and consumer behavior. Most of these assumptions have not been empirically tested; some probably cannot be tested. The authors are honest about this. For example, they ask about one normative question, "So what do buyers prefer? Current studies are inconclusive." p. 336. But the thesis of the book is that over a long enough time scale things will work out as they predict.

And, they warn, we are in trouble if they don't. "What happens if these approaches [to creating norms] . . . fail to create the norms we need? Then we have no effective way to avoid the world about which the privacy advocates warn—the world in which we leave a permanent picture of our lives that anyone can access." (p. 340) They are exactly right about the consequences of that—the loss of human potential due to unwillingness to take chances and the tendency to hold youthful nonconformity against people.

3 Opinion

Ultimately this book is an attempt to find a norm-based alternative to the notice-and-consent paradigm, which is really all we have now for commercial data processing: Force businesses to explain what they are doing with your data, and accept the clicking of the "I agree" button as a contract binding both parties. Heaven knows we need an alternative: the inadequacy of the notice-and-consent paradigm is unquestionable. As Hal Abelson, Ken Ledeen, and I wrote in *Blown to Bits* several years ago, "We have to solve our privacy problems another way." So the authors deserve a lot of credit for working out so thoroughly a theory of the alternating chess moves that a norm-based alternative would beget. And perhaps our grandchildren and our students' students will look back from the twenty-second century and point to this work as the beginning of the evolution of rational and honorable norms that solved our problems with the commercialization of our consumer data. After all, *The Academic Revolution* turned out to be a classic, even though in the short term, history overtook its title.

But for those of us who have to push product out today and teach our fellow citizens how to deal with their daily digital lives and vote on competing laws pushed by industry and consumer groups, the path laid out in Unauthorized Access seems long, winding, uncertain, and uncharted. The subtitle's "crisis," by definition, can't wait. Perhaps the book's fundamental premise will get some traction: as an alternative to notice-and-consent, the development of norms and laws that will eventually motivate buyers and sellers both to act according to appropriate trade-offs of values. Among the tragedies of the post-Snowden privacy world is that the public is distracted from the corporate behaviors on which Sloan and Wagner want to focus our attention. Not only has the glare from the government's spying programs temporarily blinded Americans to corporate surveillance, but the U.S. technology companies, now internationally mistrusted as reliable stewards of data, are victims too.

Review of [4]
Boolean Functions - Theory, Algorithms, and Applications
Yves Crama and Peter L. Hammer
Cambridge University Press, 2011
710 pages, Hardcover

Review by
Haris Aziz (haris.aziz@nicta.com.au)
NICTA and University of New South Wales, Australia

"[...] any system of propositions may be expressed by equations involving symbols x,y,z, which, whenever interpretation is possible, are subject to laws identical in form with the laws of a system of quantitative symbols, susceptible only of the values 0 and 1. But as the formal processes of reasoning depend only upon the laws of the symbols, and not upon the nature of their interpretation, we are permitted to treat the above symbols, x, y, z, as if they were quantitative symbols of the kind above described. *We may in fact lay aside the logical interpretation of the symbols in the given equation; convert them into quantitative symbols, susceptible only of the values 0 and 1; perform upon them as such all the requisite processes of solution; and finally restore to them their logical interpretation.* And this is the mode of procedure which will actually be adopted [...]—George Boole [1, Chapter V.6]. "

1 Introduction

The world has come a long way since George Boole laid the foundation of modern computers by formalizing the theory of Boolean logic. Therefore, it is not surprising that Boolean functions are one of the most fundamental and ubiquitous mathematical objects with applications and relevance to various fields such as complexity theory, logic, electrical engineering, combinatorics, and game theory. *Boolean Functions - Theory, Algorithms, and Applications* is a comprehensive study of the theoretical and algorithmic aspects of Boolean functions. Not only are the connections with various fields highlighted but work on Boolean functions in many fields within mathematics, computer science, and engineering is presented in a unified and mathematically elegant manner.

Boolean Functions - Theory, Algorithms, and Applications is a monograph with a number of contributed chapters other than the ones written by the main authors Yves Crama and the late Peter Hammer (1936 – 2006). The contributors include Claude Benzaken, Endre Boros, Nadia Brauner, Martin C. Golumbic, Vladimir Gurvich, Lisa Hellerstein, Toshihide Ibaraki, Alexander Kogan, Kazuhisa Makino, and Bruno Simeone.

2 Summary

tThe book is divided into three main parts: I. Foundations, II. Special Classes, III. Generalizations. The three main parts are followed by appendices.

Part I Part I lays down the foundation of Boolean functions, their representations, and properties.

Chapter 1 *'Fundamental concepts and applications'* starts off with introducing the two standard forms—CNF (conjunctive normal form) and DNF (disjunctive normal form). Basic representational issues are discussed. Other representations of Boolean functions such as BDDs (Binary Decision Diagrams) are also introduced.

Chapter 2 reviews one of the most fundamental problems regarding Boolean functions: solving of *Boolean equations*. Various approaches including mathematical programming to solve Boolean equations are outlined. Cook's theorem and its consequences are presented. Schaefer's famous classification theorem is also presented in which a characterization of polynomial-time solvable and NP-hard Boolean equations was first stated [3].

Chapter 3 is dedicated to two fundamental topics concerning Boolean functions: *prime implicants and minimal DNFs*. Prime implicants are the basic building blocks of a Boolean functions. The chapter also discusses the algorithmic and representations problems concerning minimal DNFs.

Chapter 4 concerns *duality theory*. Basic properties of the dual of a Boolean function are presented. This is followed by a detailed treatment of various algorithmic questions related to dual of Boolean functions. The special case of positive Boolean functions is also considered.

Part II Part II examines subclasses of Boolean functions which display interesting properties.

The part starts off with Chapter 5 on *quadratic Boolean functions*. Solving quadratic Boolean functions in CNF form is equivalent to the 2-SAT problem. The chapter also highlights how various subclasses of quadratic functions have correspondence with different classes of graphs.

The same theme is continued in Chapter 6 which covers *Horn functions*—another interesting subclass of Boolean functions. A Boolean function is in Horn DNF form if each term has at most one complemented variable. Algorithmic and structural properties of Horn functions are reviewed.

Chapter 7 examines the *orthogonal forms and shellability*. A Boolean expression is in ODNF (orthogonal disjunctive normal form) if it is in DNF and every two terms of it must be "conflicting" in at least one variable. A desirable property of ODNF is that the number of true points of an expression in ODNF can be computed efficiently. A class of boolean functions for which ODNF can be computed efficiently is the class of shellable disjunctive normal forms.

In Chapter 8, *regular Boolean functions* are covered. Regular Boolean functions are one of the most important and tractable classes of Boolean functions. They are a generalization of *threshold Boolean functions* and satisfy most of their interesting properties.

Threshold functions—also known as linearly separable Boolean functions—are surveyed in more depth in Chapter 9.

Chapter 10 covers *Read-once Boolean functions.* Read-once Boolean functions are monotone Boolean functions which can represented in a form such that each variable occurs once.

Chapter 11 titled '*Equational characterizations of special classes*' examines classes of Boolean functions and presents their characterizations by logical expressions called DNF identities.

Part III Part III is on generalizations and extensions of standard Boolean functions.

Chapter 12 surveys *partially defined Boolean functions* and natural representational and algorithmic questions regarding partially defined Boolean functions. The chapter also discusses Boolean functions from the point of view of *computational learning theory.*

Chapter 13 is on *pseudo-Boolean functions* which are real valued functions of Boolean variables.

Appendices The three main parts are follows by the appendices.

Appendix A gives a brief overview of the following concepts: decision problems; algorithms; running time; polynomial-time algorithms; the classes P, NP, coNP; polynomial-time reductions and NP-completeness; and Cook's theorem.

Appendix B presents some essential definitions from graph theory.

Finally, in Appendix C, JBool a software designed to analyze Boolean functions is presented.

3 Opinion

This is a very comprehensive monograph which should be a valuable reference for engineers, mathematicians, and computer scientists. The encyclopedic nature of book can be judged from the more than seven hundred pages and more than nine hundred references. The book does not suffer from a flaw suffered by a number of handbooks—the style and notation is consistent and the reader can seamlessly navigate from one chapter to another as one would do in a decent textbook.

The book promises to prove a valuable learning and teaching tool. Concepts are introduced in a clear manner and the text is reader-friendly and easy on the eyes. The notation used is simple and natural. All the important theorems in the field are stated in formal environments and the accompanying simplified proofs of many important results makes this a self-contained book. There are well-chosen exercise questions at the end of each chapter. A graduate course on 'Algorithmic aspects of Boolean functions' can be designed on the basis of the book.

As indicated by the title, the treatment of the topic is very algorithmic in nature. The important computational problems are stated in the formal style of Garey and Johnson [2]. Then, either known lower bound results are stated or algorithms to tackle the problem are outlined. Important algorithms are spelled out formally in easy to read pseudocode. For problems for which there exist more than one standard algorithm, a more fine-grained comparison between the algorithms is presented. For example this is the case for the four standard algorithms for quadratic Boolean equations in Chapter 5.

The book may also act as a catalyst for future research. Although it does not have a separate chapter on future research directions, interesting open problems are outlined and discussed in many chapters. Some open problems are also stated in the exercises at the end of the chapters. Outlining open problems in the text and in the exercises promises to motivate further research. This is particularly useful to young researchers and non-experts who are not necessarily aware of the state of the art in the field.

Another salient feature is that of pointing out connections with other fields and highlighting the applications of Boolean functions to logic, electrical engineering, reliability theory, game theory, voting theory, combinatorics, graphs theory, artificial intelligence, and distributed computing systems. These applications are explicitly presented just like theorems and definitions are.

The conclusion is that this elegant and comprehensive book ticks a lot of boxes. It is a great reference, suitable as a learning/teaching tool, and a valuable survey to stimulate future research. Hence, it promises to become a modern classic.

References

[1] G. Boole. *An Investigation of the Laws of Thought on Which are Founded the Mathematical Theories of Logic and Probabilities*. 1854.

[2] M. R. Garey and D. S. Johnson. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W. H. Freeman, 1979.

[3] T. J. Schaefer. The complexity of satisfiability problems. In *Proceedings of the 10th Annual ACM Symposium on Theory of Computing (STOC)*, pages 216–226. ACM Press, 1978.

Review of[5]
Additive Combinatorics
Terence Tao and Van H. Vu
Cambridge University Press, 2010
512 pages, Softcover

Review by
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1 Introduction

Additive combinatorics is a study of the combinatorial structures of sets under arithmetic operations such as addition or subtraction. Common examples of problems in this area are – for a given set A with an additive structure, what can we say about the size of the set $A + A$ (set consisting of elements which are sum of two elements in A), or does $A+A$ contain long arithmetic progressions. It combines techniques from various areas in mathematics such as basic combinatorics, graph theory, linear algebra to more advanced tools such as Fourier analysis, harmonic analysis, ergodic theory and theory of hypergraphs.

2 A chapter-wise summary of the book

1. The first chapter of this book talks about the *probabilistic method* in additive combinatorics. Probabilistic method is an important tool in combinatorics that proves existence of a certain kind of mathematical object. Often in such cases, it is difficult to come up with an algorithmic construction of the object in consideration.

   The chapter goes on to prove the Borel-Cantelli Lemma as an application and shows that additive sets of integers have a "sum-free" subset of a certain minimum size. Several basic probability estimates such as Markov's inequality, Chebyshev's inequality, Chernoff's bound and Janson's inequality and certain applications of these bounds are covered as well. Using probabilistic method, the authors also present a non-constructive proof of the Lovász Local Lemma in this chapter.

2. The second chapter discusses *sum set* estimates. A fundamental question in this area is that, if A and B are two additive sets of integers, then what can we say about the size and other properties of the set $A + B$, which consists of elements that are the sum of two elements chosen from A and B respectively.

   The chapter also talks about the *inverse sum set* problem, that is, given a bound on the size of the set $A + B$ or $A - B$, what can we say about A and B themselves. It discusses the notion of Ruzsa distance, which is a metric that roughly characterize the difference between two additive sets A and B. The authors next state the Balog-Szemerédi-Gowers Theorem, which discusses partial sums and differences of two sets, both the commutative and non-commutative versions and give some applications of it. The proof of the theorem is presented in Chapter 6.

3. In chapter 3, the authors look into the properties of sum sets of A and B, that have a underlying geometric structure, such as lattices, convex sets, arithmetic progressions, etc. The chapter proves the Brunn-Minkowski inequality which gives a lower bound on the "volume" of the sum of two geometric sets in some d-dimensional space, in terms of the volumes of the individual sets. Subsequently the authors, discuss the structure and properties of the intersection of a convex set and a lattice and show that all such intersections can be approximated by a "box" in some sense.

4. Chapter 4 discusses Fourier analytic techniques. It first develops the basic framework of Fourier analysis defining concepts such as Fourier transformation and convolution. The notion of Fourier bias is defined thereafter, which separates sets as "highly random" or "highly structured" ones. Building on this, the authors introduce the concept Bohr sets which aids in extracting "combinatorial information" about the additive set from its Fourier coefficients. They also prove Rudin's inequality which is the Fourier analytic equivalent of Chebyshev's inequality. The chapter also talks about the spectrum of additive sets and give an elegant application of it due to Bourgain and Konyagin.

5. In chapter 5, the authors revisit the questions about inverse sum set estimates. The main result of this chapter is Freiman's Theorem which states that an additive set with a small doubling constant (the ratio of $|2A|/|A|$), is contained in an arithmetic progression that is not too large than the set itself. As a tool to aid them, they discuss the concept of Frieman homomorphisms. Various versions of Frieman's theorem are proven in this chapter such as for torsion groups, torsion-free groups, and general groups.

6. In chapter 6, the authors change gears and move on to the beautiful arena of graphs. After introducing some basic terminologies in graph theory, they move to questions such as – what is the maximum size of an independent set in a general graph, in triangle free graphs?

   Later in the chapter the authors discuss applications of Ramsey's theory to additive combinatorics. The proof of Balog-Szemerédi-Gowers Theorem is given in this chapter.

7. Chapter 7 talks about the Littlewood-Offord problem (and its inverse). Suppose one chooses d elements from some additive set and considers the linear combination of these elements over the set $\{-1, 1\}$. Then what it is the largest number of repetitions among these combinations? Conversely, if this largest number is given, what can we say about the d elements then? Two different approaches to this problem is presented – one using combinatorics and the second using Fourier analysis. Interesting applications to probability theory are discussed also.

8. In chapter 8, a proof of the Szemerédi-Trotter theorem, which gives an upper bound on the number of intersections between a finite collection of points and lines. Using this result, an improved upper bound is given for the sum-product problem over real numbers (a result due to Erdös and Szemerédi). Later in the chapter this is extended to the field of complex numbers.

9. Chapter 9 talks about the use of algebraic methods in additive combinatorics. The chapter starts with the discussion of the combinatorial Nullstellensatz and applies it to obtain certain lower bounds on sum set estimates. It also explores the domain of finite fields and several problems related to sum estimates over finite fields (such as Davenport's problem, Kemnitz's conjecture). The chapter concludes with a discussion of cyclotomic fields and uses it to establish an uncertainty principle of Fourier transformations over finite fields.

10. Chapter 10 discusses a very fundamental result in additive combinatorics – namely Szemerédi's Theorem, which states that subsets of positive integers having *positive upper density* contain arbitrarily long arithmetic progressions. As a powerful tool, Szemerédi's Regularity Lemma is also introduced in this chapter. In this chapter, the existence of arithmetic progressions of length three are dealt using linear Fourier analytic techniques.

11. Chapter 11 continues from where the previous chapter ends, and goes on to prove the general version of Szemerédi's Theorem. Higher order Fourier analytic methods are developed and used for this purpose. Also other techniques such as hypergraph theory and ergodic theory, that give different proofs of Szemerédi's Theorem are also discussed.

12. Chapter 12 looks at general sets (that is without having any specific structure) and discusses results about lower bounding the length of the largest arithmetic progressions in such sets with respect to the size of the set. Several results of this flavor are discussed in this chapter.

3 Opinion

In my opinion any person who is comfortable with graduate level mathematics courses such as linear algebra, group theory, real analysis, graph theory and possessing suitable mathematical maturity will do a decent job of understanding the material covered in this book. The topics covered are mostly self sufficient. However I would have liked a more detailed treatment of Szemerédi's Regularity Lemma in Chapter 10 for the sake of completeness. The book has a huge and extensive bibliography, comprising of 388 references.

Additive combinatorics is a rapidly developing area of mathematics that strings together tools and techniques from various disciplines in mathematics. For anyone interested in this area, this is a must-have reference text. It covers a broad spectrum of needs from acting as a frequent reference to classic results in this field to important and necessary problem solving techniques. The authors of this book are pioneers in the field and do a near perfect job of collecting of essential material and put them in a very readable format.

Review[6] of
Discrete and Computational Geometry
Author: Satyan L. Devadoss and Joseph O'Rourke
Princeton University Press
355 pages, Hardcover

Review by
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1 Introduction

In *Discrete and Computational Geometry*, Satyan Devadoss and Joseph O'Rourke provide an introduction to computational geometry for undergraduates majoring in Mathematics and Computer Science. The authors assume a knowledge of discrete mathematics and some calculus and the book contains both proofs and algorithms. Often, the authors give an intuition for a solution before going into the details. As the authors claim, this book is appropriate for almost any second or third year major in mathematics or computer science.

2 Summary

The first four chapters cover what most would call the core topics of computational geometry (Polygons, Convex Hulls, Triangulations, and Voronoi diagrams). The three last chapters are less standard and give an introduction to some of the topics covered in the authors' research.

Chapter 1: Polygons. The chapter focuses on polygons in 2D. A proof sketch of the Jordan Curve Theorem for polygons is given. The authors define triangulations and show that every polygon has a triangulation. Next, the authors derive properties of triangulations, discuss counting triangulations, and consider the Art Gallery problem.

The next section considers scissors congruence in 2D. A *dissection* of a polygon P cuts P into a finite number of smaller polygons. Two polygons P and Q are scissors congruent if P can be dissected into $P_1, \ldots, P_n$ and rearranged, where each P_i can be rotated and translated, and reassembled to get Q. The section ends by proving a few lemmas for the Bolyai-Gerwein's theorem, which states that any two polygons of the same area are scissors congruent.

Chapter 2: Convex Hulls. This chapter begins with the nails and rubber band analogy to describe the convex hull of a set S of points. Two definitions for the convex hull are given: the intersection of all convex regions that contain S and the set of all convex combinations of points in S. Then, the two definitions are proven to be equivalent.

An incremental algorithm for constructing the convex hull of a set of points is presented, and compared to a mathematical proof by induction. In the analysis of this algorithm, the concept of general position is discussed. Next, additional algorithms for constructing the convex hull are presented: gift wrapping, Graham scan, divide and conquer. The final section of this chapter discusses algorithms for computing the convex hull in 3D.

Chapter 3: Triangulations. A triangulation of a point set $S \subset \mathbb{R}^2$ is 'a subdivision of the plane determined by a maximal set of noncrossing edges whose vertex set is S' [pg 58]. The incremental algorithm to construct a triangulation of a point set is given. Euler's formula is as follows: the sum of the number of vertices and faces in a planar triangulation is equal to the number of edges plus two. After proving Euler's formula, it is used to count the number of edges and triangles in a triangulation, given the number of points on the boundary of the convex hull.

Flip graphs are introduced as a way of measuring the distance between triangulations. The authors prove that the flip graph of any planar point set is connected. The authors introduce associahedra. Although this is not a common topic in a computational geometry class, it fits in well in the context of this book. Associahedra are polygons whose vertices and edges form the flip graph of a convex polygon.

As a special type of triangulation, the Delaunay triangulation is introduced. The definition, given on page 82, involving legal edges and a lexicographical ordering of triangulations, is a bit more cumbersome than the standard empty circle definition. Showing that the two definitions are equivalent is left up to the student in Exercise 3.52. To conclude this chapter, the authors discuss minimum weight triangulations, Steiner points, and pseudo triangulations.

Chapter 4: Voronoi Diagrams. The Voronoi region of a site p in S is the set of all points in the plane that are closer to p than to any other point in S. The Voronoi diagram is the collection of the boundaries of Voronoi regions, in other words, it is the collection of all points that have more than one nearest neighbor. They prove that if there are n sites, then there are at most $2n - 5$ Voronoi vertices and at most $3n - 6$ Voronoi edges.

This chapter covers three algorithms for computing the Voronoi diagram of n sites: an $O(n^2 \log n)$ algorithm that computes the Voronoi diagram as the intersection of half planes, Fortune's $O(n \log n)$ sweepline algorithm, and an $O(n^2)$ incremental algorithm.

After the duality between the Voronoi diagram and the Delaunay triangulation is explored, the chapter ends with an algorithm that computes the Delaunay triangulation by projecting each site (x, y) to a point $(x, y, x^2 + y^2)$ on a paraboloid. Then, the projection of the upper hull of the arrangement of tangent planes is the Delaunay triangulation of the original points.

Chapter 5: Curves. This chapter starts with the medial axis of a polygon P. The medial axis is the set of points that have 2 (or more) closest points on ∂P, or equivalently, it is the cut locus of ∂P. The former definition allows one to think of the medial axis as an extension of the Voronoi diagram. The chapter continues by defining the straight skeleton, a variant of the medial axis.

In this chapter, Minkowski sums are also discussed. The Minkowski sum is a way of summing two shapes. The most common Minkowski sum is a shape (in other words, an obstacle) and a circle of radius r. The Minkowski sum of the shape and the circle gives a forbidden region for the center of circular robot trying to navigate around the obstacle. The Minkowski sum is related to the convolution of the boundary curves and the winding numbers in Theorem 5.21.

In Section 5.5, the book starts to go from traditional computational geometry topics to discrete geometry. This section discusses iteratively shortening curves, first by the midpoint transformation. Then, the heat equation is applied to the curve $C(s) = (x(s), y(s))$ given as a function of the arc length s. This produces a geometric flow, or a continuous non-intersection evolution of the curve C. After this is discussed in the continuous setting, the discrete version is given followed by a section on the heat equation and the Poincaré conjecture. After discussing the Poincaré conjecture and Perlman's classification and resolution of the singularities that arise in Ricci curvature, this chapter concludes with the CRUST algorithm for curve reconstruction.

Chapter 6: Polyhedra. This chapter begins with the five Platonic solids, the only equal vertex degree, congruent face polyhedra. Classifying the 10 Archimedean solids (uses just two types of polygonal faces) is left to the student as an exercise. A polyhedron is defined as a collection of faces, edges, and vertices subject to three conditions termed: the intersection condition, the local topology condition, and the global topology condition. This is described in detail in section 6.2, concluding with a discussion of the genus of the polygonal surface.

In section 6.3, the Gauss-Bonnet theorem is defined for a smooth surface, and then discretized to work for a polyhedron. This theorem relates the integral of the Gaussian curvature (a very geometric quantity) to the Euler characteristic of the surface (a purely topological property).

Section 6.4 focuses on the proof of Cauchy's Rigidity theory, and section 6.5 looks at the nets of polyhedra. The net of a polyhedron is a connected planar layout of its faces. In this discussion, the topic of the shortest path on surfaces arises. There are three properties that a shortest path must have: no self intersections, does not go through a vertex, and there must exist a net for which this path unfolds straight. The final section of this chapter deals with geodesics on Polygons. One significant theorem presented here states that every closed surface has at least three distinct, simple, closed geodesics.

Chapter 7: Configuration Spaces. Chapter 7 begins with motion planning, in particular: finding a path (if one exists) for a polygonal object from one position on the plane to another position while avoiding m polygonal obstacles. Another problem that might arise in motion planning is to reconfigure a polygonal chain to form different shapes. As demonstrated by the knitting needles example in Figure 7.15, chains can be locked in 3D (that is, the configuration space of 3D chains is disconnected). This is justified against using the heat equation in Chapter 5 to straighten a polygonal chain in 2D. This chapter concludes with a brief overview of topological equivalence and a brief introduction to Morse theory.

3 Opinion

Discrete and Computational Geometry by Devadoss and O'Rourke covers traditional topics in computational geometry from a much different perspective than other books. Thus, we believe it is a complement to other books. The coverage of the additional topics (e.g., unfolding, associahedra, Poincaré conjecture) is unique and daring. It is remarkable that the authors were able to condense a coherent discussion of Ricci flow and the proof of the Poincaré conjecture to 2 pages. At points, however, the authors could afford to be a bit more technical, e.g., a full definition of homeomorphism instead of just the intuitive one on page 230, and using the Taylor expansion when discussing the heat equation in Section 5.6.

One stylistic difference between this book and a currently popular textbook for a computational geometry class (*Computational Geometry* by de Berg, Cheong, van Kreveld, and Overmars) is the presentation of algorithms. Devadoss and O'Rourke choose to give algorithms in paragraph form; whereas, de Berg et. al. use pseudocode. The advantage to not using pseudocode is that it can appeal to an audience without a computer science background, but the disadvantage is that there is a wider gap between the textbook and implementation of the algorithms. On the topics that are covered by both textbooks (Convex Hulls, Voronoi Diagrams, etc.), we find that the two perspectives are complementary, not contradictory.

Devadoss and O'Rourke appeal to different audiences by starting with intuition and examples before formal definitions, theorems and illustrated algorithms: e.g., the definition of the Convex Hull given in Chapter 2.

The exercises are sprinkled throughout the chapter. The problems vary in difficulty from problems that check understanding by asking the student to construct examples to starred problems that related to published research. There are even open problems that are presented in this book. In addition, a solution manual is available for instructors though the publisher.

To demonstrate the variety of exercises, we have selected a few exercises:

- (3.14) Show that every triangulation has some vertex of degree at most five.
- (4.30) Extend the [incremental Delaunay triangulation] algorithm to handle the case when the additional site p is outside the convex hull of the previous k sites.
- (Unsolved Problem 22) Find an algorithm that guarantees correct curve reconstruction from an ϵ-sample for some $\epsilon \geq 1/2$.

The book contains some very nice historical anecdotes, such as that Delaunay was a Ph.D. student of Voronoi at Kiev University. This helps to make the material seem more approachable.

In sum, we recommend this book for an undergraduate course on computational geometry. In fact, we hope to use this book ourselves when we teach such a class.

Review of[7]
Iterative Methods in Combinatorial Optimization
by Lap Chi Lau, R Ravi and Mohit Singh
Cambridge University Press, 2011
242 pages, hardcover $90.00, Paperback $40.00

Review by
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1 Introduction

This book is in the area of *combinatorial optimization*. Generally speaking, combinatorial optimization is a subject that aims to find the best object from a set of discrete and finite objects, and is one of the central areas in applied mathematics and theoretical computer science. The well-studied problems in combinatorial optimization include Traveling Salesman Problem, Minimum Spanning Tree, Knapsack Problem and so on. Combinatorial optimization is important, not only because of its natural connections to algorithms and computational complexity, but also due to its wide applications in learning theory, mathematics, software engineering, operations research and artificial intelligence.

More concretely, this book is on a specific *iterative* method used in combinatorial optimization. This method can be characterized by four steps:

- Linear Programming formulation of the given problem.
- Characterization of extreme point solution using the rank lemma.
- Iterative algorithm for constructing an integral solution from an extreme point.
- Analysis of the correctness of the algorithm, and of the optimality of the returned solution.

In addition, when the best object is hard to find, one may consider finding an object that is nearly optimal. And this scenario is the *approximate optimization*. When the iterative method is applied to approximate optimization, there are usually one or both of the two additional steps: rounding and relaxation.

The title of the book, *Iterative Methods in Combinatorial Optimization*, is somewhat misleading in the sense that this book does not survey all the iterative methods in combinatorial optimization but just one of the iterative methods.

2 Summary

This book has 14 chapters, which can be roughly categorized into 5 parts.

The first two chapters are introductions and preliminaries, and lay the foundation for the whole book. Chapter 1 uses a concrete example to show how the method works and then gives an overview of the book. Chapter 2 contains some basic knowledge in combinatorial optimization, e.g. linear programming, graphs, and submodular and supermodular functions. Chapter 2 just includes the most basic facts used in the book. Some more advanced concepts, like network matrices, will be introduced in later chapters together with the application of the iterative method.

Chapter 3, 4, and 5 survey the application of the method to three very well-studied problems, matching, spanning trees and matroids. Each chapter also includes the generalization of the method to designing approximation algorithms for some NP-hard problems. In the literature, the iterative method discussed in this book was originally used for exact optimization, and later had some novel applications in approximate optimization. This part of the book is the *core* of the whole book.

For instance, in Chapter 3, the authors first apply the iterative method to the maximum weighted matching in bipartite graph, an exact optimization problem. Then the iterative method is applied to designing approximation algorithms for two NP-hard problems, generalized assignment problem and the budget allocation problem.

Chapter 6 and 7 continue the style of Chapter 3, 4 and 5, and apply the method to some less-famous problems, including directed rooted spanning trees and submodular flow problems. This part can be seen as the generalization of the second part of the book.

The subsequent six chapters, from Chapter 8 to Chapter 13, are on applying the iterative method to much more advanced topics, like network matrices, hypergraph matching and network design. This part is not as fundamental as the first three parts, but is more relevant to the state-of-the-art results. For example, most of the results on hypergraph matching in Chapter 9 are from a very recent SODA paper (SODA 2010).

Chapter 14 is a 2-page summary of the book, and raises some interesting future research directions.

2.1 List of problems

The main purpose of the book is to exhibit the applications of the iterative method, which is simple, powerful and versatile. Here I list the problems mentioned in the book that can be solved by the iterative method. Obviously, it is unrealistic and unnecessary to read all the applications. Readers can simply choose some problems that interest them or that are relevant to their own research. Generally speaking, the results in Chapter 1, 3, 4 and 5 are fundamental and should be known to everyone interested in combinatorial optimization.

- Chapter 1: The assignment problem in bipartite graph.
- Chapter 3: The maximum weighted problem in bipartite graph, the minimum cost vertex cover problem in bipartite graph, the generalized assignment problem, and the budgeted allocation problem.
- Chapter 4: The minimum spanning tree problem, degree-bounded minimum-cost spanning tree problem.
- Chapter 5: The problem of finding a maximum weight basis in a matroid, and the problem of finding a maximum weight common independent set of two matroids, degree-bounded version of the minimum cost basis problem for matroids, unweighted k matroid intersecting problem.
- Chapter 6: The arborescence problem in directed graph, the vertex connectivity problem in directed graph, and their degree-bounded versions.

- Chapter 7: The submodular flow problem and its degree-bounded generalization.
- Chapter 8: The dual of the matroid intersection problem and the dual of the submodular flow problem.
- Chapter 9: The maximum matching problem in weighted undirected graph and its generalization to hypergraphs.
- Chapter 10: The survivable network design problem and the minimum bounded-degree Steiner network problem.
- Chapter 11: The partial vertex cover problem, and the multicriteria spanning tree problem.
- Chapter 12: The triangle cover problem, the feedback vertex set in bipartite tournaments, and the node multiway cut problem.
- Chapter 13: The hypergraph discrepancy, the rearrangements of sums in geometric setting, the single source unsplittable flow problem, bin packing problem, and the undirected Steiner tree problem.

3 Opinion

In a nutshell, this book is well-written and technically correct. It has a very clear structure in each chapter: first applying the iterative method to some exact optimization problems, and then showing the application of the iterative method in designing approximation algorithms.

I believe that the book is a good *reference* book for researchers in combinatorial optimization and approximation algorithms. The novel applications of the iterative method to a variety of settings have received great attention recently. This book, as a good survey on the applications of the iterative method, will surely inspire more results in the future. Also, since the writing style is elementary, the book is accessible to researchers with basic knowledge in linear algebra and graph theory. Additionally, at the end of each chapter, there is a bibliographic note summarizing relevant literature, and thus researchers do not need to survey themselves. Therefore, I recommend the book as a up-to-date reference for researchers.

But on the other hand, I do not think that the book is a suitable *textbook*, though the exercises after each chapter of the book are excellent and well-designed. There are two reasons. The first reason is that the topic is a little narrow, only covering one method used in combinatorial optimization. In particular, what is the relationship between the iterative method and other methods used in combinatorial optimization? The book does not cover this. The material of the book is not enough for teaching a course, and is not enough for expanding the horizon of the students. The second reason is that the style of the book is somewhat over technical in a lemma-lemma-theorem style, and is not conceptual enough for being a textbook. For instance, the definition of a crucial concept *laminar family*, used again and again in the book, is not properly highlighted.

Personally, I believe that the book is more suitable as *advanced reading material* in a course. It goes deep in the discussion of the iterative method and has a thorough survey on the applications of the method. It is known that a considerable amount of polynomial-time algorithms for certain special classes of combinatorial optimization can be unified by the theory of linear programming.

And this book just uses the theory of linear programming, and hence provides a novel perspective on the problems and algorithms in combinatorial optimization.

An additional advantage of the book is that there are free downloadable electronic copies in the homepages of the authors. So researchers who are already very familiar with this method can simply read some chapters in the book online, and see if there could be more applications of the iterative method.

Review of **Perspectives on Projective Geometry** [8]
Author: Jürgen Richter-Gebert
Publisher: Springer, 2011, 571 pp.
ISBN NUMBER: 978-3-642-17285-4, PRICE: U$ 84.95

Reviewer: S. C. Coutinho (collier@impa.br)

1 Overview

Who hasn't heard that "parallel lines meet at infinity"? What is not so well-known is that this trope is at the heart of a whole branch of mathematics: *projective geometry.*

Although the ancient Greek geometers studied geometric configurations that we now analyze using the tools of projective geometry (the best known example of which is Pappus's Theorem), the subject only began to acquire an identity after Girard Desargues's published his *Rough draft for an essay on the results of taking plane sections of a cone* in 1639. Inspired by the studies of perspective of the Renaissance painters, Desargues analyzed those properties of geometric figures that remain unchanged when the figures are projected. For instance, since ellipses, hyperbolas and parabolas can be obtained as projections of a circle from a point, one can prove many properties of those curves by checking them for the circle and showing that they are invariant under projections. Of course this does not include length, since it is possible to project a smaller circle onto a bigger one.

Desargues wrote his work in the synthetic tradition of Greek geometry, soon to be overshadowed by the analytic methods introduced by his contemporary Descartes. Actually, in a letter to Desargues, Descartes himself suggested that projective geometry would be better understood in the analytic language he had introduced. However, when projective geometry was re-discovered by Poncelet in the 19th century one of his motivations was to present a method capable of simplifying many of the proofs of geometry while avoiding complicated arguments and calculations. In order to do this, Poncelet introduced in his *Treatise on the Projective Properties of Figures* two principles. The principle of continuity allowed him to reduce problems about general conics to equivalent problems about circles; the principle of duality was used to turn statements about lines into statements about points and vice-versa.

Unfortunately these principles became an endless cause of controversy, because some of Poncelet's applications of them seemed almost impossible to formalize. These difficulties had been pointed out earlier by Cauchy, in a referee report he wrote on a memoir that Poncelet had submitted to the *Académie des Sciences* in 1820. After noting that the principle of continuity was no more than a "bold induction", Cauchy explained that all that Poncelet expected to get from that principle could be obtained by the use of analytic methods. That, however, was exactly what Poncelet wanted to avoid, so he completely ignored Cauchy's advice in his *Treatise*, published two years later.

It did not take long for mathematicians to realize the importance of projective geometry and develop those aspects of it that did not depend on the principle of continuity. Among them was Michel Chasles, a banker who retired after accumulating sufficient funds to allow him to dedicate his life to mathematics. For Chasles one of the key notions of projective geometry was that of *cross* *ratio*, which had been used by Poncelet, but actually went back at least to Pappus. Given any four points, the cross ratio is a number associated to these points that turns out to be the same even after the points are projected. This concept still plays a fundamental rôle in most modern introductions to projective geometry. It is defined in chapter 4 of the book under review.

Poncelet's opinion about analytic methods notwithstanding, mathematicians soon looked for ways to introduce coordinates in a projective plane. This was done first by Möbius, with his barycentric coordinates, and then by Plücker, whose homogeneous coordinates are the most often used nowadays. With these in place, one can write an explicit expression for the cross ratio of four points and check analytically that it is invariant under projective changes of coordinates. This expression can then be written as a quotient of determinants whose entries are the coordinates of the points.

Homogeneous coordinates were studied by Plücker's successors, among them Salmon, Hesse and Cayley. The latter was also responsible, together with Sylvester, Gordan and many others, for the development of invariant theory: the study of polynomial expressions that are unchanged under a group of transformations. Taking the expressions to be polynomials in the homogeneous coordinates of points of the projective plane and the group to be that of projective changes of coordinates, we end up with relations that hold no matter which points we choose. Among these are the Grassmann-Plücker relations, introduced in chapter 6 of Richter-Gebert's book.

Despite the fact that lengths and angles are not preserved under central projection, Cayley discovered a way to introduce measurements in the setting of projective geometry. This allowed him to prove theorems of Euclidean geometry within a projective setting. Upon reading Cayley's paper, Klein understood that this scheme did not apply only to Euclidean geometry. This led him to the so-called Cayley-Klein geometries, which also include hyperbolic geometry, elliptic geometry and the geometry of relativistic space-time. Thus, as Cayley once said, "descriptive [we now say projective] geometry is all geometry". If anything, the computer has come to confirm the truth of Cayley's saying. For example, although most users may not be aware of it, homogeneous coordinates are routinely used by computer graphics and dynamic geometry software, while bracket algebra has been applied to the mechanical proving of theorems in geometry.

2 Summary of Contents

The book under review covers all the aspects of projective geometry mentioned in the previous section. The treatment is algebraic, but not in the usual sense of writing the equations of curves in terms of the homogeneous coordinates of points—although these too are present. Instead, the author shows how long calculations with coordinates can be avoided by using invariant relations, thus fulfilling one of Poncelet's aims when he introduced projective methods. These invariants are polynomial expressions in homogeneous coordinates that remain unchanged under projective transformations. For that reason they can be written in terms of the points themselves. The simplifications obtained in this approach are very remarkable indeed.

The book opens with a chapter where many of the key ideas, to be developed later, are introduced in a very concrete setting, that of the *Pappus's hexagon theorem*. This well-known result states that given six points, of which A, B, C and A', B', C' are collinear, then the points of intersection of the lines AB' with CB', BC' with CA' and BA' with AC' are collinear. The chapter contains many proofs of this theorem (some synthetic, some analytic) as well as a number of interesting variations. In this setting one is naturally led towards the projective approach. For example, depending on the choice of points, the lines BA' and CB' can be parallel. This means that, if one is working within the context of euclidean geometry, then to the above statement of Pappus Theorem must be added some hypothesis like "when these lines intersect". The reason we did not do this is that the statement makes perfect sense in the projective plane, where any two lines always intersect. This chapter is a delightful read and a perfect introduction to the ideas behind projective geometry.

The remainder of the book is divided into three parts. The first one, called *Projective Geometry* is an introduction to the more elementary aspects of projective geometry in two dimensions. It begins with an axiomatic definition of the projective plane (chapter 2), followed by the introduction of homogeneous coordinates, which enables one to give a precise definition of Poncelet's principle of duality (chapter 3). The next two chapters introduce the cross ratio (chapter 4) and discuss its relation to the coordinates of the real projective line (chapter 5). Chapters 6 and 7 are concerned with *bracket algebra* and its application to projective geometry. Roughly speaking, a bracket is a determinant whose entries are the homogeneous coordinates of points of the projective plane (or line, or space). These brackets satisfy a number of relations that are invariant under projective transformations and give rise to the algebraic, but coordinate free, approach to projective geometry that is developed in greater detail in part II. Appropriately called *Working and Playing with Geometry*, this part begins with an application of bracket algebra to quadrilateral sets; that is, figures obtained by the intersection of four lines. Chapters 9 to 11 deal with conics and include many classic topics of projective geometry; among them, polars and primal/dual pairs (chapter 9), cross ratios, perspectivities and the theorems of Pascal and Brianchon (chapter 10) and intersections of conics with lines and other conics (chapter 11). Projective spaces of higher dimension are introduced in chapter 12, after which the subject shifts rather remarkably. The next two chapters deal with tensors, defined in terms of tables of numbers that satisfy certain rules. The author describes, in chapter 13, a nice diagram technique that simplifies many calculations with tensors, which are applied to geometry in chapter 14. Despite the obvious importance of tensors in geometry, these chapters end up as something of a dead end, for the subject is hardly mentioned elsewhere in the book. Indeed, with the next chapter we return to the applications of the bracket algebra introduced in part I. These include techniques for the automatic proof of theorems and applications of bracket algebras to the theorems of Ceva and Menelaus. One crucial point rightly emphasized by the author is that once a proof by bracket algebra has been automatically generated, it tends to be easy to check by hand. Nothing similar can be done for automatic proofs obtained using Gröbner bases, for example.

Part III is the longest of all, but its chapters share a common theme: the introduction of metric questions in a projective context. This is done via the so-called *circular points*: two points in the line at infinity whose coordinates are complex numbers. Of course these points do not belong to the real projective plane, which explains why this part of the book opens with a chapter on complex numbers, followed by one on the complex projective line. The development of Euclidean geometry using the circular points begins in chapter 18 and is further developed in chapter 19, which includes results on triangles and conics. The next five chapters give a general description of the geometries that can be defined using circular points. Called *Cayley-Klein geometries*, they introduce, within the context of projective geometry, ways by which one can measure distances and angles. These measurements are defined in chapter 20 and the transformations that preserve them are studied in chapter 21. The next two chapters cover topics such as trigonometry and circles in the general context of a Cayley-Klein geometry. Chapter 24 contains a short history of non-euclidean geometry and is followed by two chapters on hyperbolic geometry viewed as a Cayley-Klein geometry. The last chapter is called *what we did not touch* and describes a number of areas that are somehow related to the ideas developed in the book. Three of these areas (discrete mathematics, quantum computing and dynamic geometry) are closely connected to computer science. For example, the theories of matroids and oriented matroids "encapsulate the combinatorial essence of projective point and line configurations" (p. 531), while the state vector in quantum theory is determined only up to a nonzero numerical factor, so it is not really a vector, but a point in projective space. The final section of this chapter discusses applications of projective methods to dynamic geometry. This is an area to which, as a co-author of the *Cinderella* system, the author of this book has made many important contributions. Even though the section is only 10 pages long, it includes many references where the reader can find more detailed information about this subject.

3 Opinion

This lovely book offers an introduction to the key ideas of projective geometry through an algebraic, but coordinate free approach, from which one can derive shorter and more transparent proofs that can often be automated. The book is written in a leisurely way that will make it accessible to undergraduate students with a basic knowledge of algebra. If you like geometry, you will probably be hooked by the time you finish the first chapter. Remarkably, the author managed to keep, throughout the book, the same conversational style that makes the first chapter so delightful to read. Of course this does not mean that later chapters are equally elementary. However, assuming you have the pre-requisites, which are very modest, you will have no difficulty following the arguments.

There is no doubt that the author went to great lengths to write clearly and to make his book as user friendly as possible, as the well chosen illustrations, several per page, many in colour, testify. Unfortunately, his use of English is not always idiomatic. Indeed, some sentences are so obscure that one has to guess what the author wanted to say. Although this does not happen often, it is an unnecessary blemish; one that could have been easily fixed by the publishers with a little basic copy-editing, with the added advantage that it would also have removed the unorthodox spellings (like Pappos, instead of Pappus) and most of the typos. I was also left wondering if the book would not be better without the chapters on tensors. As pointed out above, they are isolated episodes, not directly related to the book's plot, and they break the flow of the story the book tells so wonderfully. Of course, these are very minor faults in a mathematics book that, despite its 570 pages, can be read with little effort and much pleasure by anyone with a very basic background in algebra and geometry.

Review of[9]
Who's #1?: The Science of Ranking and Rating
by Amy N. Langville and Carl D. Meyer
Princeton University Press, 2012
266 pages, Hardcover

Review by
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1 Introduction

This book comprises a solid introductory through intermediate text on the science of rating and ranking. The field of ranking and rating has been receiving increasing attention in the last several years due to its application in a myriad of areas from movie recommendations to selecting matchups in sports tournaments. The central question of the field is, given a list of traits about a set of entities (movies, sports teams etc.) how do we assemble a ranking (strict linear order) of the entities. There are many ways to achieve a ranking, a myriad of tuning parameters for each method, and various overall measures of accuracy. The book covers the science behind the methods and the art behind the tuning with equal gusto and provides many insightful asides to the text including examples, pitfalls, and historical notes about this rich and complex subject.

This book is a great introduction to the field (including its constituent parts in linear algebra and data mining) and contains enough depth to be used as a supplemental book in a data mining course or as a jumping off point for an interested researcher.

2 Summary

Chapter 1 provides a solid introduction to rating, ranking and other terminology that the reader will require throughout the book. This chapter also covers some of the properties of voting rules and aggregation methods such as Arrow's Theorem (which comes into play later). This chapter provides an overview of the many applications that are touched on throughout the book and frames much of the later discussion of methods in the context of these motivating examples.

Chapters 2 through 8 are similar in structure so I will discuss them together. Each of these chapters explores a method for producing a rating (a numerical value for each object) and/or a ranking (an ordered listing of the set of objects). These middle chapters deal with, respectively, the Massy Method, the Colley Method, Keener's Method, ELO, Markov Methods, the authors own Offense/Defense method, and several rank aggregation methods. Each of these chapters provides enough mathematical grounding for one versed in linear algebra to easily follow along through the derivation of the method. The particular method of the chapter is illustrated with a running example (a football team ranking problem). In fact, all the application of these methods is done on sports team scenarios using point differentials and other numerical statistics to provide the input.

Since the same problem is used in most of the chapters the authors are able to show where and under what conditions some of the methods in the chapters disagree. This provides the authors several interesting points and keeps the reader engaged through some of the longer mathematical derivations. Each of the methods is discussed in enough detail for one to implement the particular method in something like MatLab or Python with numpy (though no code is provided).

Chapter 9 provides the first break from the implementation chapters that come before it. This chapter deals mainly with the concept of point spreads in sports gambling. Informally, a spread is how many points a team is expected to win or lose by. Most sports betting is done "against the spread," which include a certain number of points by which a team has to win. Historical background and an analysis of the point spread is provided. The authors explain why a rating system is not intended to guess the spread and, in fact, it is not feasible to devise a rating system that would do such a thing. It is an extremely interesting chapter and seems to be placed in order to quash the non-mathematical gambler's dream of devising a rating or ranking system to guess points spreads (and provide infinite riches) right as it would be beginning to develop after reading all the chapters before.

Chapter 10 extends the ideas presented in chapters 2-8 to work with preference data. While the explanation in the earlier chapters required numerical data, many times one only has access to preference data (e.g., I prefer movie A to movie B) without a notion of relativity (e.g., team A beat team B by 50 points) as used in the previous chapters. The authors explain how to extend the methods discussed in earlier chapters for use with this type of data and show application of these methods for the rating problem for Netflix movies.

Chapter 11 is all about ties. Since the primary focus of the book is on designing ranking algorithms for sports teams, ties are something that one must contend with. Up to this point in the book the authors assume that the input data does not contain ties. In this chapter the authors discuss different ways of cleaning the input data so that this assumption is upheld. Interestingly, the authors also introduce the use of induced ties and their ability to improve ranking systems. An induced tie can be thought of as follows: in a game like basketball, the game cannot end in a tie. However, particularly close games (decided by just a point or so) may have gone either way. The authors show that introducing ties into the input data can actually improve the resulting ranking of several methods.

Chapter 12 focuses on how to add weights to input data. One can imagine that in many applications certain games (for sports) or preferences (for movies) are more important than others and should be considered more important. In this chapter, the authors provide an extremely detailed how to manual on adding weights to all the methods discussed earlier and an analysis of the impact weighting can have on the final list. The authors develop their treatment of weights with a good amount of mathematical detail and clearly demonstrate the impact of considering weights in rating and ranking problems.

Chapter 13 is a relatively short remark about sensitivity analysis when it comes to rating and ranking methods. The authors discuss how to test the various discussed methods for sensitivity and remark about the pitfalls of several of the discussed methods. This chapter seems to be more of a warning of things to look out for than an instruction manual like some of the other chapters.

Chapter 14 and 15 discuss aggregation methods for combining multiple lists. Chapter 14 nicely ties the section to voting and the uses of voting for aggregating multiple lists. The chapter shows the benefits and pitfalls of combining lists under different methods and lays the groundwork for the discussion of ensemble methods in the next chapter. Chapter 15 feels like it dives much more into the authors own research than previous chapters. Much of the discussion of Chapter 15 is about how to formulate the list aggregation problems as various kinds of linear programs (and methods for solving these linear programs). Much of the latter half of Chapter 15 deals with particular relaxation techniques for the LP's put forth in the first half. While the ideas this chapter puts forward are interesting (ensemble methods are how the Netflix prize was won so it is an important topic) much of the mathematics and discussion seems to be targeted over the heads of the intended readers.

Chapter 16 delves into the topic of comparing lists. The chapter discusses the normal statistics for comparing ranked lists (Kendall's Tau and Spearman's Rho) as well as the derivation of a new list statistic devised by the authors. I enjoyed the author's discussion of the development of their comparison statistic and I think students interested in ranking and rating would be well served by understanding how and why some of the research decisions were made.

Chapter 17 and 18 close out the book and provide, respectively, a list of available data sources and many references for further reading. The authors have compiled a very nice list of places to get sports data and additional methods that more advanced students may be interested in pursuing. These last two chapters alone make the book worth the price to beginning graduate students!

3 Opinion

This is a well written book that could be used in many different ways (as the authors point out in their introduction). The first several chapters are setup in a way such that advanced high school students or undergraduate university students could use the techniques as practical application exercises in a mathematics (linear algebra) course. Allowing students to implement any of the matrix methods described in chapters 2-8 with real data would be an interesting assignment or stand alone section worthy of inclusion in an introductory linear algebra course. Additionally, the number of interesting asides and application notes would keep many students interested in the subject if this was used as a supplementary text in a mathematics course.

The book contains no exercises and some of the mathematical treatments are not in-depth enough for this to stand alone as a textbook. However, the book does cover enough ground, and enough depth, about ranking and rating to compliment advanced undergraduate or early graduate courses that wish to cover ranking, rating, applications of linear algebra, or light data mining aspects. This book would be a wonderful introduction to any advanced undergraduate or beginning graduate student wishing to research in the area thanks to its good organization, interesting examples, and very nice bibliography.

Overall this is a very nice, well written book that could be use in multiple ways by a wide variety of audiences.

Review of[10]
Boosting : Foundations and Algorithms
by Robert E. Schapire and Yoav Freund
544 pages, 2012, $40.00 MIT press
Reviewer:Shiva Kintali `kintali@gmail.com`

1 Introduction

You have k friends, each one earning a small amount of money (say 100 dollars) every month by buying and selling stocks. One fine evening, at a dinner conversation, they told you their individual "strategies" (after all, they are your friends). Is it possible to "combine" these individual strategies and make million dollars in an year, assuming your initial capital is same as your average friend ?

You are managing a group of k "diverse" software engineers each one with only an "above-average" intelligence. Is it possible to build a world-class product using their skills ?

The above scenarios give rise to fundamental theoretical questions in machine learning and form the basis of *Boosting*. As you may know, the goal of machine learning is to build systems that can adapt to their environments and learn from their experience. In the last five decades, machine learning has impacted almost every aspect of our life, for example, computer vision, speech processing, web-search, information retrieval, biology and so on. In fact, it is very hard to name an area that cannot benefit from the theoretical and practical insights of machine learning.

The answer to the above mentioned questions is *Boosting*, a elegant method for driving down the error of the combined classifier by combining a number of weak classifiers. In the last two decades, several variants of Boosting are discovered. All these algorithms come with a set of theoretical guarantees and made a deep practical impact on the advances of machine learning, often providing new explanations for the existing prediction algorithms.

Boosting : Foundations and Algorithms, written by the inventors of Boosting, deals with variants of *AdaBoost*, an adaptive boosting method. Here is a quick explanation of the basic version of AdaBoost.

AdaBoost makes iterative calls to the base learner. It maintains a distribution over training examples to choose the training sets provided to the base learner on each round. Each training example is assigned a weight, a measure of importance of correctly classifying an example on the current round. Initially, all weights are set equally. On each round, the weights of incorrectly classified examples are increased so that, "hard" examples get successively higher weight. This forces the base learner to focus its attention on the hard example and drive down the generalization errors.

AdaBoost is fast and easy to implement and the only parameter to tune is the number of rounds. The actual performance of boosting is dependent on the data.

2 Summary

Chapter 1 provides a quick introduction and overview of Boosting algorithms with practical examples. The rest of the book is divided into four major parts. Each part is divided into 3 to 4 chapters.

Part I studies the properties and effectiveness of AdaBoost and theoretical aspects of minimizing its training and generalization errors. It is proved that AdaBoost drives the training error down very fast (as a function of the error rates of the weak classifiers) and the generalization error arbitrarily close to zero. Basic theoretical bounds on the generalization error show that AdaBoost overfits, however empirical studies show that AdaBoost does not overfit. To explain this paradox, a margin-based analysis is presented.

Part II explains several properties of AdaBoost using game-theoretic interpretations. It is shown that the principles of Boosting are very intimately related to the classic min-max theorem of von Neumann. A two-player (the boosting algorithm and the weak learning algorithm) game is considered and it is shown that AdaBoost is a special case of a more general algorithm for playing a repeated game. By reversing the roles of the players, a solution is obtained for the online prediction model thus establishing a connection between Boosting and online learning. Loss minimization is studied and AdaBoost is interpreted as an abstract geometric framework for optimizing a particular objective function. More interestingly, AdaBoost is viewed as a special case of more general methods for optimization of an objective function such as coordinate descent and functional gradient descent.

Part III explains several methods of extending AdaBoost to handle classifiers with more than two output classes. AdaBoost.M1, AdaBoost.MH and AdaBoost.MO are presented along with their theoretical analysis and practical applications. RankBoost, an extension of AdaBoost to study ranking problems is studied. Such an algorithm is very useful, for example, to rank webpages based on their relevance to a given query.

Part IV is dedicated to advanced theoretical topics. Under certain assumptions, it is proved that AdaBoost can handle noisy-data and converge to the best possible classifier. An optimal boost-by-majority algorithm is presented. This algorithm is then modified to be adaptive, leading to an algorithm called BrownBoost.

Many examples are given throughout the book to illustrate the empirical performance of the algorithms presented. Every chapter ends with *Summary* and *Bibliography*, mentioning the related publications. There are well-designed exercises at the end of every chapter. Appendix briefly outlines some required mathematical background.

3 Opinion

Boosting book is definitely a very good reference text for researchers in the area of machine learning. If you are new to machine learning, I encourage you to read an introductory machine learning book (for example, *Machine Learning* by Tom M. Mitchell) to better understand and appreciate the concepts. In terms of being used in a course, a graduate-level machine learning course can be designed from the topics covered in this book. The exercises in the book can be readily used for such a course.

Overall this book is a stimulating learning experience. It has provided me new perspectives on theory and practice of several variants of Boosting algorithms. Most of the algorithms in this book are new to me and I had no difficulties following the algorithms and the corresponding theorems. The exercises at the end of every chapter made these topics much more fun to learn.

The authors did a very good job compiling different variants of Boosting algorithms and achieved a nice balance between theoretical analysis and practical examples. I highly recommend this book for anyone interested in machine learning.