HS Projects

Math Projects

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4) What changes if $A \subseteq \mathbb{Z}$ or $A \subseteq \mathbb{Q}$ or $A \subseteq \mathbb{D}$.

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3) Other domains like $\{a + b\sqrt{2} \colon a, b \in \mathbb{Z}\}$.

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For all a, b relatively prime the set of primes $\equiv a \pmod b$ is infinite.

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Might get into other questions about domains.

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2) Apply P to $\mathbb{D}$ and note where it fails.
3) See what domains P does apply to.

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These are called **Sicherman Dice** and you can buy them on amazon.

Funky Dice: Setting

If you roll two 6-sided dice then $\mathrm{Prob}(2) = \frac{1}{36}$, $\mathrm{Prob}(7) = \frac{1}{6}$, and the other probs can be worked out.

This was all based on both dice being labelled $(1, 2, 3, 4, 5, 6)$.

Is there a **different** way to label the dice where you get the same probabilities?

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Question For which d is there a relabelling of two d-sided dice that gives the same probabilities as the standard labeling? We call these **Sicherman dice**.

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Is there a way to relabel a 4 sided die and a 6 sided die to get the same probabilities as the usual 4-sided and 6-sided dice?

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5) Combine this with loaded dice (Gasarch and Kruskal have written the main paper on that).

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What if you have 6 points? 7 points?

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What happens if the dimensions are not powers of 2?

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Asymmetric: $n \times m$ board.

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How can you prove $\pi \neq \frac{22}{7}$ using relatively elementary means?

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Variants Prove other irrationals are not the close to some rationals.

Crypto Projects

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Example (A AND B) or (B AND C AND D) determine s.

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Our question: how long a sequence can A and B form.

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Key Find cases where the hard protocol is better than that easy protocol.

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3) Come up with protocols that are harder to break.

AI Projects

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There are 12 coins. One is counterfeit and is either heavier or lighter. You have a balance scale so you can take x coins on one side and x coins on the other and either it balances or one is heavier. How many weightings do you need to find which coin is counterfeit?

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There does not seem to be any math structure to use.

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Spoiler's goal is to maximize the number of weightings.

Counterfeit Coins Problem: Project

Lets use AI on it!

How?

Set it up as a game. We look at $c = 2$.

Weighter picks two sets of x coins to balance.
Spoiler gives an answer that is consistent with having 2 counterfeits.
Repeat this until Weighter knows what the counterfeit coins are.
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Train two AI's (perhaps neural nets) to do well at these games and see if the number-of-weightings converges to a number that might be the truth.

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Bonus You get to learn more about the classical versions of this problem.

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For which (a, b) does ONE win? This has not been researched much.

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There are many other games involving numbers on a board that one can study. **Example** One number on the board and the playes remove factors of it, lose if you get 0.

Circle Crossing Game: Setting and Project

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The usual: AI, see who wins, variants, blah blah.