Cheat Sheet for Ramsey Ordering Talk

Notation 0.1 $a, n \in \mathsf{N}$. A, B, X are sets. A, B are ordered.

1. $[n] = \{1, \ldots, n\}$.
2. $\binom{X}{a}$ is the set of a-sized subsets of X.
3. $A \equiv B$ mean that there is a bijection from A to B.
4. $A \equiv_o B$ mean that there is an order preserving bijection from A to B.

Def 0.2 $a \in \mathsf{N}$. X a set. COL : $\binom{X}{a} \rightarrow [< \omega]$. $H \subseteq X$ is *homog* if COL restricted to $\binom{H}{a}$ is constant (i.e., takes on only ONE value).

We state 2-ary inf Ramsey Thm in an odd way to make a point.

Thm 0.3 *$\forall d \in \mathsf{N}$, $\forall$ COL : $\binom{\mathsf{N}}{2} \rightarrow [d]$, $(\exists H \equiv \mathsf{N})[H$ homog$]$.*

What is we color $\binom{\mathsf{Z}}{2}$ and care about the ordering: Is the following true?
Let $d \in \mathsf{N}$. Let COL : $\binom{\mathsf{Z}}{2} \rightarrow [d]$. $\exists$ $H \equiv_o \mathsf{Z}$ such that H is homog.

NO. Counterexample. We color $\mathsf{Z} - \{0\}$ for convenience.

$$\mathrm{COL}(x,y) = \begin{cases} 0 & \text{if } x, y \geq 1 \\ 1 & \text{if } x, y \leq -1 \\ 2 & \text{if } x, y \text{ differ in sign} \end{cases} \tag{1}$$

Need new definition of homog and a new question.

Def 0.4 $a \in \mathsf{N}$. X ordered set. COL : $\binom{X}{a} \rightarrow [< \omega]$. A set $H \subseteq X$ is *X-h-homog* if COL restricted to $\binom{H}{a}$ takes on $\leq h$ values.

Def 0.5 $a_1, a_2 \in \mathsf{N}$. X_1, X_2 ordered set. COL : $\binom{X_1}{a_1} \times \binom{X_2}{a_2} \rightarrow [< \omega]$. A set $(H_1, H_2) \equiv_o (X_1, X_2)$ is *(X_1, X_2)-h-homog* if COL restricted $\binom{H_1}{a_1} \times \binom{H_2}{a_2}$ takes on $\leq h$ values.

Q: X ordered set. $a \in \mathsf{N}$. What is least h such that $\forall$ COL : $\binom{X}{a} \rightarrow [< \omega]$, $\exists$ an X-h-homog set? Denote this $T(\binom{X}{a})$.

Q: X_1, X_2 ordered sets. $a_1, a_2 \in \mathsf{N}$. What is least h such that
$\forall$ COL : $\binom{X_1}{a_1} \times \binom{X_2}{a_2} \rightarrow [< \omega]$ $\exists$ (X_1, X_2)-h-homog set? Denote this $T(\binom{X_1}{a_1} \times \binom{X_2}{a_2})$.