Referee's Report for
On the two-colour disjunctive Rado Number for the equations $\sum_{i=1}^{m-2} x_i + ax_{m-1} - x_m = c - j,\ j = 1, 2$
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1 Final Decision

2 Abstract

You say that *for some range of values of* c_1 *and* c_2. That is fine. but what about a? Will you determine $\mathcal{R}_d(L)$ for all $a \in \mathsf{Z}$? This should be specified in the abstract.

3 Introduction

1. Page 1. There is a lot missing in your discussion of Rado's theorem which sets the stage for your work. In particular there are two kinds of questions to ask and you don't distinguish them. I will discuss this for the case of one equation.

   - **Rado's Theorem for Single Equations:** Let $a_1, \ldots, a_m \in \mathsf{Z}$ and let $E(x_1, \ldots, x_m) = \sum_{i=1}^{m} a_i x_i$. the following are equivalent

     (a) for all r there exists n such that for all r-colorings of $\{1, \ldots, n\}$ there is a monochromatic solution to E. (Note: the value of n from the standard proof is enormous and the smallest n that works is thought to be much smaller.)

     (b) some non-empty subset of $\{a_1, \ldots, a_m\}$ sums to 0.

   - Say b above is false. Let M be the max sum of all nonempty subsets of $\{a_1, \ldots, a_m\}$. Let p be the least primes bigger than M. There is an $M-1$-coloring of N with no mono solution. But what about smaller values like $M-2$? Or as in this paper, 2. So the question is, for equations that do not satisfy b, what happens with 2-coloring.

2. Page 2. Were Schaal & Zinter the first paper to consider the case where a constant (c) was allowed?

3. Page 2. Theorem 1, Proposition 2, Theorem 3: You have not defined $\mathcal{R}(c)$. I suspect its the least n such that, for all 2-colorings of $\{1,\ldots,n\}$, there is a monochromatic solution to

$$\sum_{i=1}^{m-2} x_i + ax_{m-1} - x_m = c.$$

However, if thats the case then $\mathcal{R}$ should be a function of a, c, m.

4 Results for $\sum_{i=1}^{m-2} x_i + ax_{m-1} - x_m = c_i$, $i = 1, 2$

1. Page 3. Proposition 4. You begin with $(1,\ldots,a)$ but never use that. Replace the first two sentences with

   Let $a, \lambda, n \in \mathsf{N}$ *such that* $a \geq 3$, $n \geq 1$ *and* $\lambda \geq a-1$.

2. Page 3. Theorem 5 is far less interesting than it appears.

   - The coloring of $[1, k-1]$ that has no monochromatic solution has no solution at all.
   - The proof that any 2-coloring of $[1, k]$ has a monochromatic solution is $x_1 = \cdots = x_m = k$. So the x_i's are all the same color since they are all the same number.

   I am *not* suggesting you remove Theorem 5. I suggest that you propose (or solve) open questions that ask for a more interesting solution. For example

   Open Question: Investigate a variant of $\mathcal{R}$ where there is an condition that the monochromatic solution can't be all the same number.

3. The notation is inconsistent and confusing. Note the following.

   - The title of Section 2 is equation we are dealing with is

     $$\text{Result for} \sum_{i=1}^{m-2} x_i + ax_{m-1} - x_m = c_i$$

     You use the index i twice and in different ways. That is the i in $\sum_{i=1}^{m-2}$ and the i in c_i are different.

- Proposition 4 uses $\sum_{k=1}^{n} x_k + ax_{n+1} = N$.

  You should not use k as an index since later k is used in $c = k(a + m - 3)$.

- Theorem 5. The statement uses c_i with $i = 1, 2$. Do you also use $\sum_{i=1}^{m-2}$? Implicitly since you discuss $\mathcal{R}(k_1, k_2)$. And you *do* use that sum in the proof itself. So again you are using i two ways.

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4. Page 3. Equations 2a and 2b use c_1', c_2', a. After the equation you say what x_1', c_2, a' are. But a' was never ued in Equations 2a and 2b. I think you meant to have a' instead of a in Equations 2a and 2b. Please check before making the change.

5. Theorem 6 is hard to read. That cannot be helped; however, you should have before Theorem 6 a statement and proof of an actual example of the theorem.

6. Theorem 6. c_j' is defined but never used.

7. Theorem 6. Lower Bounds. Readability. You need to have titles in boldface or italics to seperate the cases like this:

   Cases 1 and 4

   Cases 2 and 3

8. Theorem 6. Lower Bounds. Readability. You need to have titles in boldface or italics to seperate the cases like this:

5 Open Problems Section (You should have one)

1. Theorem's 5 and 6 cover many but not all cases of the equation

$$\text{Result for } \sum_{i=1}^{m-2} x_i + ax_{m-1} - x_m = c_i$$

   You should have an open problems section where you state

- The simplist case that is open.
- The set of cases that is open (if that is easy to state).
- Your opinion if you have one.

2. Your results are about linear equations with coefficients all 1's except for one -1 and one a. What about other linear equations?

3. Your results are all about 2 colors. What about 3 or more?