Fall 2000 **CMSC 451: Homework 0, PRACTICE** Clyde Kruskal

Do NOT hand in.

Problem 1. For each pair of expressions (A, B) below, indicate whether A is O, o, Ω, ω, or Θ of B. Note that zero, one or more of these relations may hold for a given pair; list all correct ones.

	A	B
(a)	n^{100}	2^n
(b)	$(\log n)^{12}$	$\sqrt{n}$
(c)	$\sqrt{n}$	$n^{\cos(\pi n/8)}$
(d)	10^n	100^n
(e)	$n^{\log n}$	$(\log n)^n$
(f)	$\log(n!)$	$n \log n$

Problem 2. Prove by induction

$$\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}.$$

Problem 3. Assume every node of a tree with n nodes has either 0 or 3 children. How many leaves does the tree have? Prove your answer.

Problem 4. Solve the recurrence

$$T(n) = \begin{cases} T(n/5) + 2 & n > 1 \\ 3 & \text{otherwise} \end{cases}$$

assuming n is a power of 5. Show your calculations.

Problem 5. A coin is tossed n times, each time with an independent probability p of coming up heads and $1 - p$ of coming up tails. Let H be the number of heads occurring. What is

(a) $E[H]$, the expected number of heads?

(b) $V[H]$, the variance of H?

(c) the standard deviation of H?

(d) the probability that $H > 2$?

Show your calculations.

Problem 6. Assume that you are given a list of n numbers (some possibly negative) and we wish to find, not only the maximum contiguous sum but also the sublist itself. Give an efficient algorithm for solving this problem. What is its time?