Fall 2000 **CMSC 451: Homework 2** Clyde Kruskal

Due at the start of class Thursday, October 12, 2000.

Problem 1. Let $G = (V, E)$ be a directed graph. The *reversal* of G is a graph $G^R = (V, E^R)$ where the directions of the edges have been reversed (i.e. $E^R = \{(i,j) | (j,i) \in E\}$).

(a) Assuming that G is represented by an adjacency matrix $A[1..n, 1..n]$, give an $O(n^2)$-time algorithm to compute the adjacency matrix A^R for G^R.

(b) Assuming that G is represented by an adjacency list Adj$[1..n]$, give an $O(n+e)$-time algorithm to compute an adjacency list representation of G^R.

Problem 2. Consider a rooted DAG (directed, acyclic graph with a vertex – the root – that has a path to all other vertices). Give a linear time $(O(|V| + |E|))$ algorithm to find the length of the longest simple path from the root to each of the other vertices. (The length of a path is the number of edges on it.)

Problem 3. Let $G = (V, E)$ be an undirected graph. Give an $O(n + e)$-time algorithm to compute a path in G that traverses each edge in E exactly once in each direction.

Problem 4. Do Exercise 23.5-3 on page 494 of CLR. Justify your answer.

Problem 5. An *uttering point* in an undirected graph is a vertex whose removal separates the graph into at least three connected components. Give an efficient algorithm to find all of the uttering points of a graph. Both describe your algorithm briefly in English, and write the pseudo-code. How fast is your algorithm?

Problem 6. A *Hamiltonian path* in a graph is a path that visits every vertex exactly once.

(a) Show that if a graph has a Hamiltonian path then it does not have an uttering point.

(b) Show that the converse is not true. That is, it is *not* the case that, if a graph does not have a Hamiltonian path then it has an uttering point. (Just give an example.)